

# The Pre-FOMC Announcement Drift and Private Information: Kyle Meets Macro-Finance

Chao Ying\*

University of Minnesota

## Job Market Paper

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### Abstract

This paper studies the private information explanation for the time-series pre-FOMC drift through risk reduction. Using transaction-level data, I document the informed trading is in the same direction of the realized returns in the 24-hour window before FOMC announcements, coinciding with the pre-FOMC uncertainty reduction. I integrate Kyle's (1985) model into a standard consumption-based asset pricing framework where the market makers are compensated for the risk of assets' fundamentals. Observing aggregate order flow, they update the belief about the marginal utility-weighted asset value, which resolves uncertainty gradually and results in an upward drift in market prices before announcements. I demonstrate that there is a strictly positive pre-FOMC drift if and only if the market makers require risk compensation.

*Keywords:* Pre-FOMC announcement drift; informed trading; private information; risk compensation; uncertainty resolution

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# 1 Introduction

Lucca and Moench (2015) document the substantial stock market returns before the Federal Open Market Committee (FOMC) announcements. They find that the pre-FOMC drift of the S&P 500 index is on average 49 basis points during the 24-hour window preceding FOMC announcements, which corresponds to about 80% of the annual realized excess returns in the stock market. However, the hours and days before FOMC meetings fall into the “blackout period,” a time when policymakers and Fed staff refrain from discussions of monetary policy information.<sup>1</sup> It provides a notable challenge to standard asset pricing theory, which predicts equity returns should be earned *at* the announcements when uncertainty is resolved from the public news, *rather than ahead*.

Some papers have shown suggestive evidence that the pre-FOMC drift may come from private information before announcements. Cieslak, Morse, and Vissing-Jorgensen (2019) and Vissing-Jorgensen (2020) provide a history of leak discussions in FOMC documents and argue that systematic information leakage drives the pre-FOMC announcement drift. In addition to information leakage, market participants may generate their proprietary information by collecting data related to FOMC announcements.<sup>2</sup> In this paper, I study the private information explanation for the time-series pre-FOMC drift through risk reduction.

Empirically, I provide asset-market-based evidence that supports the presence of private information and insider trading before FOMC announcements. First, Hu, Pan, Wang, and Zhu (2020) find the significant and systematic reduction of market uncertainty (measured by the CBOE VIX index) during the same 24-hour window before FOMC announcements. Second, sorting the FOMC days via 24-hour uncertainty reduction before announcements into terciles, I find that only the group with substantial uncertainty reduction preceding announcements are associated with the positive pre-FOMC drift. Third, to measure informed trading, I calculate the order imbalances, defined as the difference between buyer- and seller-initiated trading volumes divided by total trading volume. When the uncertainty reduces before FOMC, the abnormal order imbalances are 1.85-2.17% higher in the direction of the

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<sup>1</sup>The blackout period begins at the start of the second Saturday (midnight) Eastern Time before the beginning of the meeting and ends at midnight Eastern Time on the next day after the meeting.

<sup>2</sup>For example, Kurov, Sancetta, Strasser, and Wolfe (2017) show that proprietary information permits forecasting announcement surprises in some cases.

realized return in the 24-hour window before FOMC announcements.

To understand the above features of the financial markets, I build a model that the pre-FOMC announcement drift is earned as risk reduces through insider trading. I integrate Kyle's (1985) model into a standard consumption-based asset pricing framework such that the market makers are compensated for the risk of assets' fundamentals. The equity premium is realized with uncertainty reduction prior to announcements since insider trading reveals private information. Characterizing the equilibrium price and insider trading by closed-form, I establish a strictly positive pre-FOMC announcement drift if and only if the market makers are *risk-compensated*.

Since FOMC announcements provide information about the macro-economy, the market makers require risk compensation in assets' fundamentals before the announcements. To provide macroeconomic conditions for the market makers' pricing decisions, I develop a continuous-time equilibrium model in which the aggregate economic growth is driven by a latent state variable and an i.i.d. component (short-run shocks). The investors cannot observe the latent variable directly and update the belief by observing the aggregate endowment when there is no announcement. The competitive market makers set the price, which equals the marginal utility weighted payoffs through the stochastic discount factor (SDF) determined from the above economy. The counter-cyclical SDF applies extra discounting to payoffs positively correlated with utility. Thus, the asset market requires a premium for such payoffs relative to risk-free returns.

The Fed has some extra knowledge regarding the economy, which should be revealed through periodic FOMC announcements. However, the insider knows the underlying information before announcements and trades to maximize the expected terminal profit, understanding the order affects the price. Meanwhile, the liquidity traders have random, price-inelastic demands as in the standard Kyle model. By observing aggregate order flow, the market makers update the estimation of asset payoffs as well as the SDF simultaneously such that their uncertainty is resolved.

Here are some implications of the equilibrium with the risk-compensated market makers. First, the equilibrium price is a submartingale, instead of a martingale in the standard

continuous-time Kyle-type models.<sup>3</sup> The intuition is as follows. Because of risk compensation, on average, the price of risky assets increases as uncertainty is resolved through insider trading before announcements. The slope of the expected pre-FOMC drift is the negative variance between the innovation of the SDF and the asset value. I prove a strictly positive pre-FOMC drift if and only if the market makers are compensated for the risk of assets' fundamentals. The positive excess return leads to positive average order imbalances before announcements. In the meantime, to entice the insider to trade and release information early, the market makers have incentives to set the price impact that increases on average, implying the submartingale property of the price impact. Second, due to the average upward drift in market prices, the market makers rationally anticipate that the insider would trade positively on average to chase that premium. The insider also has to consider the additional price impact from uncertainty resolution via her trading, which is unique in this model. In the equilibrium, instead of being zero, the expected order rate is determined by the ratio of the pre-FOMC drift's slope to Kyle's lambda. Additionally, as the market makers converge to be risk-neutral, the limit of the equilibrium is well defined and converges to the traditional Kyle model. The equilibrium implications indicate that this paper provides a microfoundation of how private news diffuses drive positive pre-FOMC drift in a standard microstructure framework.

Since uncertainty is not always reduced before FOMC meetings, I generalize the benchmark model so that the insider may not be better informed, and the market makers assess whether the insider has private information or not. In addition to updating their belief of asset payoffs and the SDF, the market makers estimate the probability that insiders have private information simultaneously, which are solved by a nonlinear filtering technique. Conditional on the insider being informed, the closed-form equilibrium price is a submartingale as the benchmark when the market makers are risk-compensated. The growth rate of the expected pre-FOMC announcement drift and the expected insider's order rate are time-varying, caused by the dynamics of the probability estimate. The pricing rule is nonlinear and stochastic, which drives price volatility, market depth, and price response to be stochastic. I calibrate the model so that stochastic pricing dynamics are consistent with both the

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<sup>3</sup>See Back (1992), Back and Pedersen (1998), Li (2013), and Collin-Dufresne and Fos (2016), etc.

*level* and the *trend* of the time-varying 24-hour pre-FOMC announcement drift in the data.

Before concluding, I demonstrate that other asset market evidence around FOMC announcements is consistent with the model's predictions. First, empirically the pre-FOMC drift is stronger when the uncertainty reduces more before announcements, coinciding with the risk-based explanation. Second, to maximize her profits, the insider faces a tradeoff between uncertainty and liquidity. She wants to trade later such that the market uncertainty is higher. However, she can not trade too late since she needs substantial liquidity trading to hide her position. It explains the pre-FOMC drift's timing that occurs 24 hours before announcements when private information is probably known way before. Third, I document the market uncertainty decreases significantly only before announcements with press conferences since April 2011, which explains the two distinctive patterns to equity returns found in Boguth, Gregoire, and Martineau (2019) and agrees with my model. Fourth, the sign of the pre-FOMC drift in the model depends on whether the asset is risky or a hedge. The average of time-varying betas of nominal bond is close to zero from 1996 to 2019, resulting in the absence of the pre-FOMC drift in fixed income instruments.

## **Related literature**

The paper relates to several strands of the literature. First is the large body of work investigating the impact of asymmetric information on asset prices and price impacts, seminal examples of which include Kyle (1985) and Back (1992).<sup>4</sup> I build on this literature by exploring the implications of the risk-compensated market makers, which have been largely ignored in the literature.<sup>5</sup> The equilibrium price in this model is a submartingale instead of a martingale since the resolution of uncertainty is associated with the realizations of the premium. Meanwhile, the market makers rationally anticipate that the insider would trade positively on average to chase that premium. The insider also has to consider this additional price impact from uncertainty resolution when she trades, which is unique in my model. The dy-

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<sup>4</sup>A short list is Foster and Viswanathan (1996), Back and Pedersen (1998), Back, Cao, and Willard (2000), Caldentey and Stacchetti (2010), Li (2013), Collin-Dufresne and Fos (2016), Back, Crotty, and Li (2018), Drechsler, Moreira, and Savov (2018), Dai, Wang, and Yang (2019), Crego (2020).

<sup>5</sup>While Subrahmanyam (1991) considers risk-averse market makers under CARA utility in a one-period Kyle model, there is no pre-announcement drift since the fundamental risk in the news is not priced due to CARA utility.

dynamic game between the market makers and the insider results in a positive expected order rate from the insider that contrasts this model to the literature. The limit of the equilibrium is the traditional Kyle model as the market makers converge to be risk-neutral, establishing the link between this paper and the literature.<sup>6</sup>

This paper exploits the main insight in macro-finance literature, which addresses the importance of macroeconomic conditions to account for asset prices. Starting from the equity premium puzzle in Mehra and Prescott (1985) and Hansen and Jagannathan (1991), the literature has explored a wide range of alternative preferences and market structures to account for the equity market dynamics.<sup>7</sup> However, very little attention has been paid to the pre-FOMC drift, which corresponds to about 80% of the annual realized excess returns in the stock market.<sup>8</sup> Integrating Kyle's (1985) model into a standard consumption-based asset pricing framework, I establish a strictly positive pre-announcement drift if and only if the market makers are risk-compensated in the presence of insider trading. Therefore, this paper also fills the gap between macro-finance literature and microstructure literature related to Kyle (1985).

My paper contributes to the broader literature on the premium around FOMC announcements.<sup>9</sup> Savor and Wilson (2013) find a significant equity market return on days with major macroeconomic announcements.<sup>10</sup> Lucca and Moench (2015) document the substantial stock market return during the 24-hour period preceding FOMC announcements. Theoretically, Ai and Bansal (2018) provide a revealed preference theory for the macroeconomic announcement premium in a representative agent economy.<sup>11</sup> Given the market microstructure in

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<sup>6</sup>This paper is also connected to inventory models, such as Stoll (1978) and Ho and Stoll (1981). They consider a risk-averse market maker that holds undesired positions and requires the compensation in terms of a bid-ask spread. However, these papers abstract from information asymmetries. Vayanos (2001) studies a model with risk-averse market makers that the large trader's private information is about her endowment shocks rather than the fundamental value of asset.

<sup>7</sup>See reviews in Cochrane (2017).

<sup>8</sup>Section 7 talks about the details of other explanations, including Hu, Pan, Wang, and Zhu (2020), Laarits (2019), and Cocoma (2020).

<sup>9</sup>See more discussions in section 2.1, section 6, and section 7.

<sup>10</sup>Ernst, Gilbert, and Hrdlicka (2019) find the FOMC appears to stand out from the other macroeconomic announcements in all their results, which has the largest point estimates for the concentration of the equity premium.

<sup>11</sup>To account for the pre-FOMC announcement drift, Ai and Bansal (2018) assume the contents of announcements are communicated to the public a few hours before the pre-scheduled announcements, which leads all investors to receive informative signals before FOMC announcements. However, FOMC members refrain

Kyle's model, this paper accounts for both the level and the trend of the pre-FOMC drift in the presence of private information when the market makers are risk-compensated. The endogenous uncertainty reduction before announcements through insider trading agrees with the evidence documented in Hu, Pan, Wang, and Zhu (2020). This paper can be extended to study other pre-event drifts documented in the literature. A large group of papers treats average abnormal positive excess returns before events as evidence of insider trading and tests the market liquidity implications inspired by Kyle (1985), such as other macroeconomic announcements (Kurov, Sancetta, Strasser, and Wolfe (2017)), mergers and acquisitions or earnings announcements (Keown and Pinkerton (1981), Penman (1982), and Meulbroek (1992)), and other events (Sinha and Gadarowski (2010), Agapova and Madura (2011), Collin-Dufresne and Fos (2015)). However, in the standard Kyle-type model, the expected average excess return before announcements is zero since the market makers are risk-neutral. Therefore, this paper provides a general theoretical framework for other pre-event drifts out of private information as long as the risk of the event is priced in the pricing kernel.

The rest of the paper is organized as follows. Section 2 provides empirical evidence that indicates the presence of private information before FOMC news. To provide macroeconomic conditions for the market makers' pricing decisions, I present the standard consumption-based asset pricing framework in section 3. Section 4 extends Kyle's (1985) model to the case where the market makers are risk-compensated and characterizes the equilibrium price and insider trading. In section 5, I generalize the benchmark model that the insider may not be informed, and the market makers assess whether the insider has private information or not. Section 6 tests the further implications of the model. I discuss the challenges of the private information explanation mentioned in the literature and talk about other explanations in

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from discussions of monetary policy during this period, which implies that it is almost impossible that the public systemically receives information before announcements. Besides, the implication of this assumption is not consistent with other empirical facts upon FOMC announcements. For example, there are still monetary policy surprises (see Nakamura and Steinsson (2018)), huge trading volume as well as significant realized volatility (see Lucca and Moench (2015), Bollerslev, Li, and Xue (2018), and Ying (2020)) after FOMC announcements. None of these can happen if all investors know the information before announcements. Based on the generalized risk sensitivity in Ai and Bansal (2018), Ai, Bansal, Im, and Ying (2018) and Wachter and Zhu (2018) develop quantitative models of the announcement premium under a representative agent. Ying (2020) measures the impact of FOMC announcements on disagreement in a general equilibrium model with heterogeneous beliefs.

section 7. Section 8 concludes. The Appendix contains additional details on the empirical analysis and the proof.

## 2 Empirical Evidence

In this section, I summarize the potential sources of private information before FOMC announcements and discuss the suggestive evidence shown in the literature. After that, I provide asset-market-based evidence that supports the presence of private information before FOMC news. First, I present that the market uncertainty (measured by the VIX index) decreases significantly and systematically during the same window as the pre-FOMC drift, as documented in Hu, Pan, Wang, and Zhu (2020). Second, sorting the FOMC days via 24-hour uncertainty reduction before announcements into terciles, I find that only the group with substantial uncertainty reduction preceding announcements are associated with the positive pre-FOMC drift. Third, I document there is only significant insider trading (measured by order imbalances) when uncertainty decreases before announcements, which is consistent with the private information explanation.

### 2.1 Sources of private information before FOMC meetings

The literature has provided suggestive evidence of private information before announcements. The private information may be obtained by leakage. Cieslak, Morse, and Vissing-Jorgensen (2019) propose that information about the Federal Reserve’s unexpected accommodating monetary policy is leaked ahead of the FOMC announcement, which causes a pre-announcement equity market rally. Vissing-Jorgensen (2020) provides a history of leak discussions in FOMC documents to show that the FOMC itself expresses frequent concerns about leaks. For example, the leakage led to the resignation of Richmond Fed President Lacker following admission of her involvement in the leak of confidential FOMC information to Medley Global Advisers in 2012. Finer (2018) documents an abnormal number of NYC taxi rides to the district of liberty street certain times before FOMC announcements. Besides, the private information may come from the accidental information leakage—“word-of-mouth” interpretation of information diffusion, which has been well studied in the liter-



ature of takeovers (see Keown and Pinkerton (1981), Jarrell and Poulsen (1989), Meulbroek (1992), and Augustin et al. (2015)).

The other potential source of private information is through proprietary data collection related to FOMC announcements. Given the huge market attention to FOMC announcements, to infer what the Fed knows, institutional investors have strong motivations to obtain the information that the Fed observed and keep updating the prediction model of monetary policy from historical data.<sup>12</sup> Kurov, Sancetta, Strasser, and Wolfe (2017) support this explanation by finding that proprietary information permits forecasting announcement surprises in some cases.

## **2.2 The average cumulative VIX change and return before FOMC announcements**

To capture the changes of market expectations in a timely manner, I use the CBOE VIX index, which is a model-free measure of implied volatility computed from the S&P 500 index option prices. For the intraday returns, I obtain transaction-level data on S&P 500 index (SPX). The sample period is from January 1996 to December 2019. During this period, there are in total 187 scheduled releases of FOMC statements. Except 9 of them, other releases are either around 2:15 p.m. ET (before April 2011) or 2:00 p.m. ET (after April 2011).<sup>13</sup> Therefore, I follow Lucca and Moench (2015) and focus on the 2 p.m.-to-2 p.m. pre-FOMC window, which should not contain any announcement information if there is no private information revealing.

Figure 1 shows the average cumulative VIX change and average cumulative return on the S&P 500 index around FOMC announcements. The solid line of the right panel represents the mean pointwise cumulative intraday percentage return of the SPX over a four-day window from the market open of the day ahead of scheduled FOMC meetings to the day after. Over the window from Day -3 through the beginning of Day -1, the average VIX in-

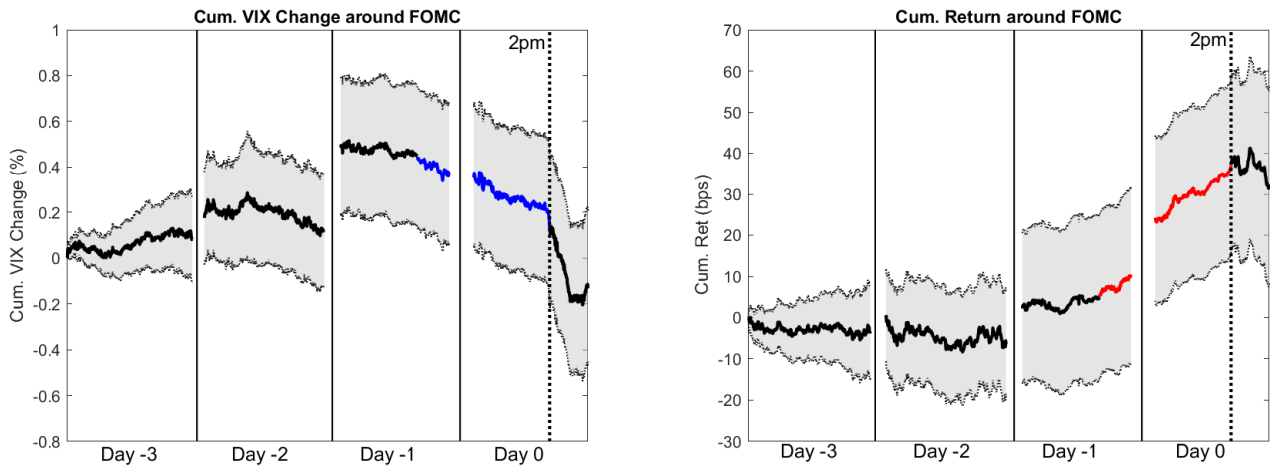
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<sup>12</sup>For example, institutional investors can hire the talented, well-trained economists who help the Fed process and interpret all the information being released, as discussed in Nakamura and Steinsson (2018).

<sup>13</sup>8 of the 9 exceptions are released around 12:30 p.m. ET from April 2011 to December 2012. Another exception happened at 11:30 a.m. ET on March 26, 1996 because of the Chairman's other duties. The results hold robustly without these releases.

creases due to the huge uncertainty of the upcoming FOMC news. While as shown in Table 2, the VIX decreases 0.3% with a  $t$ -stat of -3.4 during the 24-hour period preceding FOMC announcements, which is consistent with Hu, Pan, Wang, and Zhu (2020).<sup>14</sup> Meanwhile, the cumulative pre-FOMC drift over the same window is on average 33.2 basis points, which is statistically significant at the 1% level.

**Figure 1:** The average cumulative VIX change and return around FOMC announcements



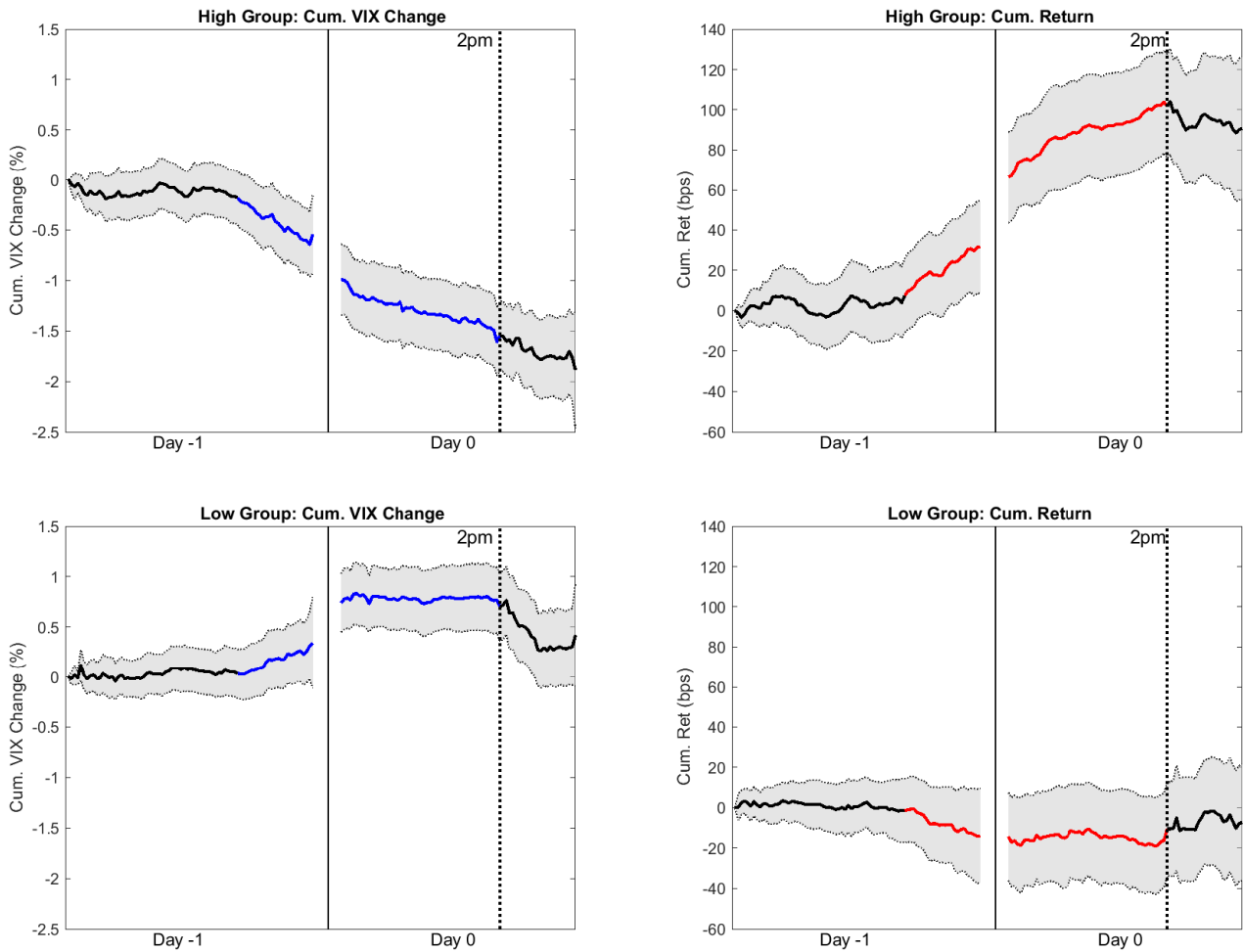
This figure shows the average cumulative VIX change and average cumulative return on the S&P 500 index on four-day windows from 1996 to 2019. The solid line of the left (right) panel is the average cumulative VIX change (average cumulative return of the SPX) from 9:30 a.m. ET on three days prior to scheduled FOMC announcements to 4:00 p.m. ET on days with scheduled FOMC announcements (labeled as Day 0). The blue (red) solid line indicates the VIX change (cumulative return of the SPX) on the 2 p.m.-to-2 p.m. pre-FOMC window. The gray shaded areas are pointwise 95% confidence bands around the average. The sample period is from January 1996 to December 2019. The dashed vertical line is set at 2:00 p.m. ET, when FOMC announcements are typically just released or 15 minutes before the release.

The significant reduction of VIX index shows the systemic uncertainty reduction out of the revealing of FOMC news preceding announcements. However, while it is common for FOMC members to express their views about macroeconomic developments or monetary policy issues in meetings or conversations with members of the public, they refrain from these discussions in the week before FOMC meetings. The 24-hour pre-FOMC window is

<sup>14</sup>The resolution of uncertainty occurs in two stages on the FOMC day, before and after the announcement. I focus on the pre-announcement reduction of VIX, which accounts for about 50% of the total decrease around FOMC meetings.

part of the blackout period. Therefore, the significant uncertainty reduction in this window indicates the potential presence of private information before FOMC announcements.

**Figure 2:** Classifications of FOMC meetings: sort on the reduction of uncertainty



This figure shows the average cumulative return on the S&P 500 index on two-day windows for the high group and low group, respectively, where I sort the reduction of uncertainty in the 24-hour pre-FOMC window into terciles. The blue (red) solid line indicates the VIX change (cumulative return of the SPX) on the 2 p.m.-to-2 p.m. pre-FOMC window.

### 2.3 Classification of FOMC announcements via uncertainty reduction

I sort the FOMC days by their reduction of uncertainty during the 24-hour window before announcements into terciles. Figure 2 plots the cumulative VIX change and the cumula-

tive return around FOMC meetings for the high-reduction group and low-reduction group, separately. The high-reduction group's VIX index decreases 1.459% significantly over the 2 p.m.-to-2 p.m. pre-FOMC window, which is associated with a deeper pre-FOMC drift (94.4 basis points) than the average FOMC results, as shown in Table 2. By contrast, the low-reduction group's VIX index increases instead of decreases before announcements and there is no positive pre-announcement drift.

This classification demonstrates that not all the FOMC announcements are the same—only the ones with uncertainty reduction preceding announcements are associated with the positive pre-FOMC drift. This is consistent with the private information explanation through risk reduction that

Later I show this is consistent with the full model in section 5 that the pre-announcement drift only occurs when the insider is informed.

In addition to the uncertainty reduction prior to FOMC announcements, Abdi and Wu (2018) find that corporate bond returns and trade directions before FOMC announcements predict the pre-FOMC stock market returns. Park (2019) shows that speculators' spread trades in bond futures have predictive information about future FOMC meetings and concludes that private information plays a key role in explaining the pre-FOMC drift. All of these asset-market-evidence indicates the presence of private information before FOMC news.

## 2.4 Measurement of informed trading

Informed trading is not directly observable. Following the microstructure literature, I measure informed trading activity by the order imbalance in the testing security defined as  $\frac{B-S}{B+S}$ , where  $B$  ( $S$ ) is the aggregate buyer-initiated (seller-initiated) trading volume. I use two measures of imbalance,  $OIN$  and  $OID$ , where volume is defined as number of trades and dollar trading volume, respectively.

Bernile, Hu, and Tang (2016) argue that the E-mini S&P 500 futures (E-mini) is the best testing security for the pre-FOMC drift.<sup>15</sup> Following their paper, I classify trading volume

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<sup>15</sup>Here are their three reasons. First, the asset underlying ES contracts is the S&P 500 index. Second, E-mini is available for trading almost 24 hours on the Globex electronic platform of the CME. Third, E-mini is

of E-mini as buyer- or seller-initiated using the tick rule. Specifically, a transaction is classified as buyer-initiated (seller-initiated) if the transaction price is higher (lower) than the last different transaction price. For each time window, the corresponding order imbalance is the difference between the total buyer- and seller- initiated volumes divided by the total trading volume.

To study the pre-FOMC drift, I examine the 24-hour window preceding FOMC announcements,  $[-24H, 0]$ . Informed trading leads to the diffusion of private information and uncertainty reduction before announcements. Therefore, for each announcement, I construct a categorical variable,  $UR$ , that equals positive one (negative one) when the uncertainty reduces, and the cumulative return is positive (negative) over the 24-hour window.  $UR$  is zero otherwise.

In Figure 3, for each FOMC announcement, I plot the order imbalance based on number of trades ( $OIN$ ) and dollar volume ( $OID$ ) in the 24-hour window before FOMC. When there is uncertainty reduction ( $UR = \pm 1$ ), most order imbalances tend to be in the direction of the realized return before announcements and large in magnitude. While when the uncertainty does not decrease before announcements ( $UR = 0$ ), the order imbalance is smaller and largely random. Table 3 compares the average order imbalances in  $[-24H, 0]$  window when uncertainty reduces before FOMC announcements ( $UR = \pm 1$ ) and when uncertainty does not reduce before FOMC announcements ( $UR = 0$ ). The average order imbalances are significantly positive on days with pre-FOMC uncertainty reduction, which is consistent with the positive pre-FOMC drift. The difference between the average  $OIN$  ( $OID$ ) of group  $UR = \pm 1$  and group  $UR = 0$  is 1.99% (2.85%) with a t-stat of 5.11 (5.24) in the 24-hour window before FOMC, which is not only statistically but also economically significant. The trading activity across uncertainty-reduced and non-uncertainty-reduced announcements show notable differences, supporting the presence of informed trading before announcements when there is uncertainty reduction.<sup>16</sup>

Next, I assess the statistical significance of these differences. To measure *abnormal* trading activities on announcement days, I also calculate the order imbalances in the same trading hour windows of non-announcement days in the 21 trading days prior to the current

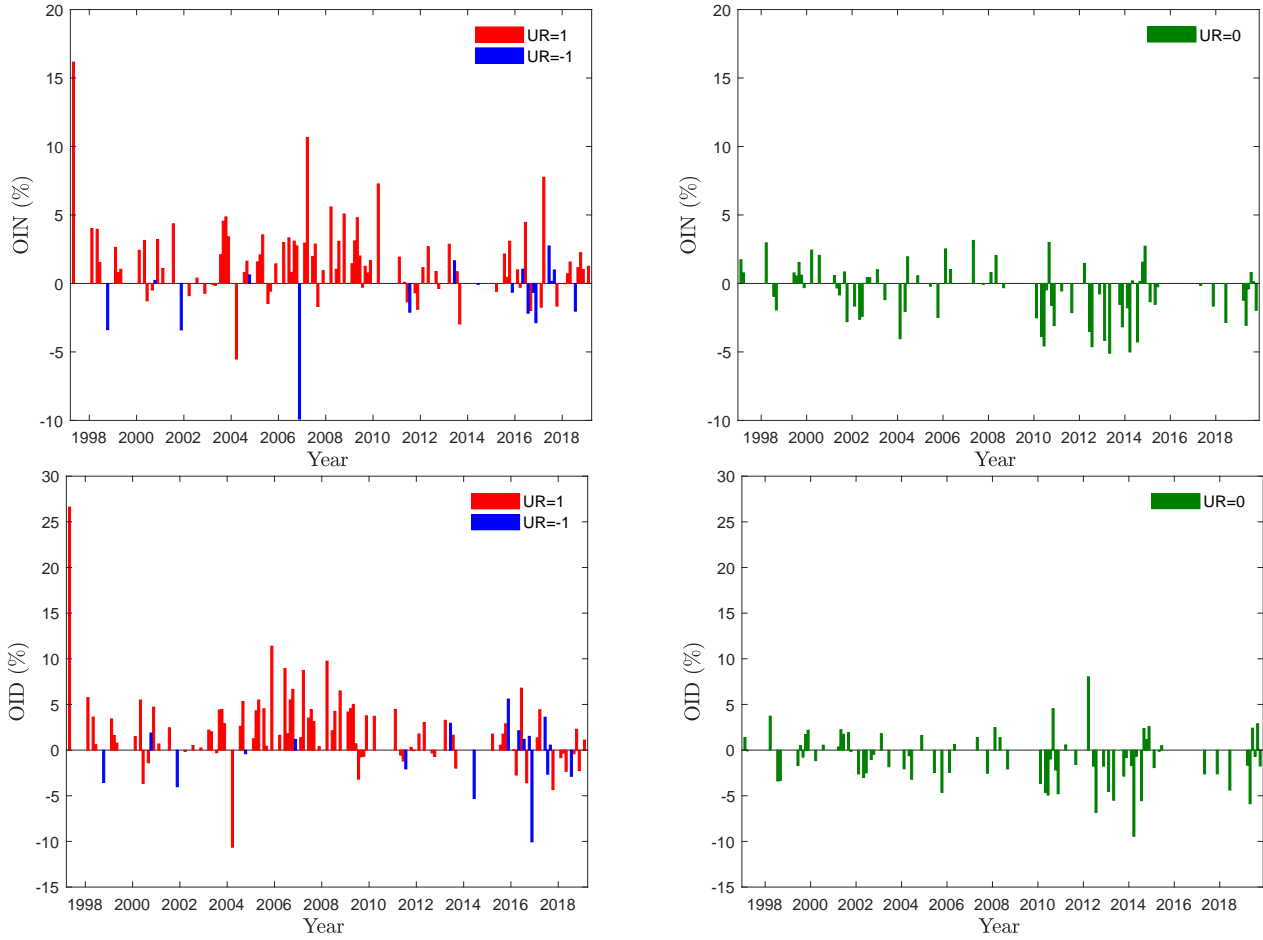
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substantially more liquid comparing to other products.

<sup>16</sup>The same pattern holds for other pre-event windows, such as  $[-12H, 0]$  and  $[-24H, -12H]$ .

FOMC announcement. I regress the two order imbalance measures,  $OIN$  and  $OID$  on the announcement indicator ( $ANN$ ) and the uncertainty-reduced indicator ( $UR$ ). Table 4 reports the ordinary least squares (OLS) coefficient estimates.

**Figure 3:** Measurement of informed trading: order imbalance



This figure shows the order imbalance based on number of trades ( $OIN$ ) and dollar volume ( $OID$ ) of E-mini in the 24-hour window before FOMC announcements. Red (blue) bars represent the order imbalance when the uncertainty reduces, and the cumulative return is positive (negative) over the 24-hour window, i.e.,  $UR = 1$  ( $UR = -1$ ). Black bars represent the average order imbalance when the uncertainty does not reduce in the 24-hour window before FOMC announcements.

The  $UR$  coefficient estimates in Columns 1 ( $OIN$ ) and 2 ( $OID$ ) are positive, and statistically significant, with  $t$ -stat of 5.61 and 4.60, respectively. When there is uncertainty reduction, on average, in the 24-hour window, the number and dollar volume of market orders executed in the direction of the realized pre-FOMC return exceed those in the wrong direc-

tion by 1.85% and 2.17% of the total volume, respectively. As shown in Columns 3-6, the similar pattern holds for other pre-event windows, such as  $[-24H, -12H]$  and  $[-12H, 0]$ . It provides robust evidence of informed trading in the 24-hour window before FOMC announcements when there is uncertainty reduction, agreeing with the private information explanation for the pre-FOMC drift.<sup>17</sup>

### 3 The standard asset pricing framework

To provide macroeconomic conditions for the market makers' pricing decisions, in this section, I present a standard consumption-based asset pricing framework that the economy's growth rate is not observable. Later on in section 4, I introduce the key elements in the microstructure literature, including the insider, liquidity traders, and the market makers.

#### 3.1 Physical setup of the model

There are a large number of identical infinitely lived households in the economy. I assume that the consumption of the representative agent,  $C_t$ , follows

$$\frac{dC_t}{C_t} = m_t dt + \sigma_C dB_{C,t}, \quad (1)$$

where  $m_t$  is a continuous-time AR(1) process (an Ornstein-Uhlenbeck process) unobservable to the agent in the economy. The law of motion of  $m_t$  is

$$dm_t = a_m (\bar{m} - m_t) dt + \sigma_m dB_{m,t}. \quad (2)$$

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<sup>17</sup>Bernile, Hu, and Tang (2016) find evidence consistent with informed trading only until about 30 minutes before scheduled FOMC announcements. After I replicate their paper, I extend their results to  $[-24H, 0]$  and find the coefficient estimates of their surprise indicator  $SUR$  is not significantly different from zero. The main difference is that we have different definitions of when the insider trading may happen before FOMC announcements. They think the insider trading may only happen when the surprise of FOMC news is large so that the insider can make huge profits. While my explanation is based on the risk-reduction channel instead of unexpected news, the insider can make huge profits from the uncertainty of the FOMC news, even the mean of the news is the same as expected. This is consistent with the model's prediction, as shown in section 4.

The standard Brownian motions  $B_{C,t}$  and  $B_{m,t}$  in equations (1) and (2), respectively, are independent.

At time 0, the agent's prior belief about  $m_0$  can be represented by a normal distribution. Although  $m_t$  is not directly observable, the agent can use two sources of information to update belief about  $m_t$ . First, the realized consumption path contains information about  $m_t$ , and second, at pre-scheduled discrete time points  $T, 2T, 3T, \dots$ , additional signals about  $m_t$  are revealed through announcements. For  $n = 1, 2, 3, \dots$ , I denote  $s_n$  as the signal observed at time  $nT$  and assume  $s_n = m_{nT} + \varepsilon_n$ , where  $\varepsilon_n$  is i.i.d. over time, and normally distributed with mean zero and variance  $\sigma_s^2$ .

Given the information structure, the posterior distribution of  $m_t$  is Gaussian and can be summarized by its first two moments. I define  $\hat{m}_t = E_t[m_t]$  as the posterior mean and  $q_t = E_t[(m_t - \hat{m}_t)^2]$  as the posterior variance, respectively, of  $m_t$  given information up to time  $t$ . For  $n = 1, 2, \dots$ , at time  $t = nT$ , the agent updates her belief using Bayes' rule:

$$\hat{m}_{nT}^+ = q_{nT}^+ \left[ \frac{1}{\sigma_s^2} s_n + \frac{1}{q_{nT}^-} \hat{m}_{nT}^- \right]; \quad \frac{1}{q_{nT}^+} = \frac{1}{\sigma_s^2} + \frac{1}{q_{nT}^-}, \quad (3)$$

where  $\hat{m}_{nT}^+$  and  $q_{nT}^+$  are the posterior mean and variance after announcements, and  $\hat{m}_{nT}^-$  and  $q_{nT}^-$  are the posterior mean and variance before announcements, respectively. A special case is that the announcements can completely reveal the information about  $m_t$ , which means,  $\sigma_s^2 = 0$ . Therefore,  $\sigma_s^2$  measures the transparency of FOMC announcements.

In the interior of  $(nT, (n+1)T)$ , the agent updates her belief based on the observed consumption process using the Kalman-Bucy filter:

$$d\hat{m}_t = a_m [\bar{m} - \hat{m}_t] dt + \frac{q(t)}{\sigma_C} d\tilde{B}_{C,t}, \quad (4)$$

where the innovation process,  $\tilde{B}_{C,t}$  is defined by  $d\tilde{B}_{C,t} = \frac{1}{\sigma_C} \left[ \frac{dC_t}{C_t} - \hat{m}_t dt \right]$ . The posterior variance,  $q(t)$  satisfies the Riccati equation:

$$dq(t) = \left[ \sigma_m^2 - 2a_m q(t) - \frac{1}{\sigma_C^2} q^2(t) \right] dt. \quad (5)$$



### 3.2 Preferences and the SDF

I assume that the representative agent is endowed with a Kreps-Porteus preference with risk aversion  $\gamma$  and intertemporal elasticity of substitution  $\psi$ .<sup>18</sup> In continuous time, the preference is represented by a stochastic differential utility, which can be specified by a pair of aggregators  $(f, \mathcal{A})$  such that in the interior of  $(nT, (n+1)T)$ ,

$$dV_t = [-f(C_t, V_t) - \frac{1}{2}\mathcal{A}(V_t)\|\sigma_V(t)\|^2]dt + \sigma_V(t)dB_t \quad (6)$$

I adopt the convenient normalization  $\mathcal{A}(v) = 0$  and denote  $\bar{f}$  the normalized aggregator. Under this normalization,  $\bar{f}(C, V)$  is:

$$\bar{f}(C, V) = \frac{\rho}{1-1/\psi} \frac{C^{1-1/\psi} - ((1-\gamma)V)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)V)^{\frac{1-1/\psi}{1-\gamma}-1}}. \quad (7)$$

The case of  $\psi = 1$  is obtained as the limit of (7) with  $\psi \rightarrow 1$ :

$$\bar{f}(C, V) = \rho V [(1-\gamma) \log C - \log [(1-\gamma)V]].$$

Because announcements typically result in discrete jumps in the posterior belief about  $m_t$ , the value function is typically not continuous at announcements. Given our normalization of the utility function, for  $t = nT$ , the pre-announcement utility and post-announcement utility are related by:

$$V_t^- = \mathbb{E}_t^- [V_t^+],$$

where  $\mathbb{E}_t^-$  represents expectation with respect to the pre-announcement information at time  $t$ .

In the above setup, I can show that the value function of the representative agent takes the form

$$V(\hat{m}, t, C_t) = \frac{1}{1-\gamma} H(\hat{m}, t) C_t^{1-\gamma},$$

---

<sup>18</sup>In this paper, I focus on the recursive utility. I can extend it to other preferences that satisfy generalized risk sensitivity defined in Ai and Bansal (2018).

for some twice continuously differentiable function  $H(\hat{m}, t)$ . The HJB equation and the corresponding boundary conditions for  $H(\hat{m}, t)$  can be found in Appendix. Given the utility of the representative agent, the state price density, denoted  $\{\pi_t\}_{t=0}^\infty$  can be characterized by the following lemma.

**Lemma 1.** *For  $n = 1, 2, 3 \dots$ , in the interior of  $((n-1)T, nT)$ ,  $\pi_t$  is a continuous diffusion process with the law of motion*

$$\frac{d\pi_t}{\pi_t} = -r(\hat{m}, t) dt - \sigma_\pi(\hat{m}, t) d\tilde{B}_{C,t},$$

where  $r(\hat{m}, t)$  is the instantaneous risk-free interest rate and  $\sigma_\pi(\hat{m}, t)$  is the market price of risk. At announcements,  $t = nT$ ,  $\pi_t$  is discontinuous, and the announcement stochastic discount factor (A-SDF) is given by

$$\Lambda_{t,t+\Delta}^* = \frac{[H(\hat{m}_{t+\Delta}, t+\Delta)]^{\frac{1}{\psi}-\gamma}}{[\mathbb{E}_t(H(\hat{m}_{t+\Delta}, t+\Delta))]^{\frac{1}{\psi}-\gamma}}. \quad (8)$$

For convenience, I focus on unit IES  $\psi = 1$ , which results in

$$H(\hat{m}_t, t) = e^{-\frac{\gamma-1}{a_m+\rho}\hat{m}_t+\mathcal{H}(t)} \equiv e^{-\gamma^A\hat{m}_t+\mathcal{H}(t)},$$

where  $\gamma^A \equiv \frac{\gamma-1}{a_m+\rho}$ .<sup>19</sup> Moreover, the A-SDF  $\Lambda_{t,t+\Delta}^*$  is counter-cyclical if and only if the agent has early resolution of uncertainty, i.e.,  $\gamma > \frac{1}{\psi}$ , which is equivalent to  $\gamma^A > 0$  when  $\psi = 1$ .

Given the above A-SDF, any asset with value  $A(\hat{m}_t^+, t^+)$  upon announcements  $t = nT$  will be valued by weighting the payoffs through investors' future marginal utility and taking expectations:

$$P(\hat{m}_t^-, t^-) = \mathbb{E} \left[ \frac{H(\hat{m}_t^+, t^+)}{\mathbb{E}_t^-[H(\hat{m}_t^+, t^+)]} A(\hat{m}_t^+, t^+) \mid \hat{m}_t^-, q_t^- \right], \quad t = nT.$$

When there is no private information prior to announcements, the FOMC information is only revealed upon announcements, which results in reductions of uncertainty, and realizations of the equity premium *at, rather than ahead of*, the announcements, as shown in Ai and

<sup>19</sup>The proof in the appendix provides the formula of SDF for a general IES. All the main results hold under the general IES, which are available upon request.

Bansal (2018), Ai, Bansal, Im and Ying (2018), and Wachter and Zhu (2018).

## 4 The benchmark: risk-compensated market makers

To capture the pre-FOMC announcement drift as well as the uncertainty reduction before announcements, I introduce the insider trading into this macroeconomic framework. I extend Kyle’s (1985) model (in the continuous-time formulation given by Back (1992)) to allow that market makers are compensated for the risk of assets’ fundamentals, where the market makers estimate the discounted value of the risky asset and the A-SDF simultaneously before announcements.

### 4.1 Model setting

The insider in the stock market observes the signal of announcements  $s_n = x_{nT} + \varepsilon_n$  at  $t = nT - 1$ , which happens before FOMC announcements.<sup>20</sup> Thus, she knows the underlying expected growth rate  $\hat{m}_{nT}$  and the value of the  $A(\hat{m}_{nT}, nT)$  earlier than other investors in the market. In addition to the insider, there are liquidity traders who have random, price-inelastic demands. All orders are market orders and are observed by all market makers. Denote by  $Z_t$  the cumulative orders of liquidity traders through time  $t$ . The process  $Z$  is assumed to be a Brownian motion independent of  $\varepsilon_n$ , which has mean zero and variance  $\sigma_z^2$  (per unit of time). Let  $X_t$  denote the cumulative orders of the insider and set  $Y = X + Z$ .

Given the macroeconomic conditions defined in last section, from equation (8), the market makers’ A-SDF at  $t = nT - 1$  is:<sup>21</sup>

$$\Lambda_{nT-1, nT}^* = \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}_{nT-1} [H(\hat{m}_{nT}, nT)]'} \quad (9)$$

<sup>20</sup>In section 6.2, I discuss that even the insider is probably informed way before, it is optimal that she starts to trade around the highest average market uncertainty, i.e., 24 hours before announcements shown in Figure 1.

<sup>21</sup>Similar to Ai and Bansal (2018), I assume that aggregate consumption does not instantaneously respond to the FOMC announcements. This assumption is well motivated because the announcement returns are realized in a 24-hour window before FOMC and the consumption response, if any, at this frequency is not likely to be significant enough to rationalize the magnitude of the premium.

and later on they update the estimate of the A-SDF based on the observed cumulative order flow before announcements. The market makers, who are competitive, set the price at time  $t \in [nT - 1, nT]$  as

$$P_t = \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y]} A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right], \quad (10)$$

where I denote by  $\mathcal{F}_t^Y$  the information filtration generated by observing the entire past history of aggregate order flow  $Y$  (which I denote by  $Y^t = \{Y_s\}_{s \leq t}$ ). At  $t = nT - 1$ , the market makers have a prior that the expected growth rate upon announcements  $\hat{m}_{nT}$  is normally distributed  $N(\hat{m}_{nT-1}, \Delta Q)$  where  $\Delta Q = q_{nT-1} - q_{nT}$ , as other agents in the economy (except the insider).<sup>22</sup> Here  $\frac{1}{q_{nT}} = \frac{1}{\sigma_s^2} + \frac{1}{q_{nT-1}}$  from Bayes' rule. I follow the literature to assume the value of  $A(\hat{m}_{nT}, nT)$  follows a log-normal distribution.<sup>23</sup> More specifically, I specify  $\log A(\hat{m}_{nT}, nT) = \beta \hat{m}_{nT} + N(nT)$ , where  $\beta > 0$  measures how the asset value moves with respect to the fundamental.<sup>24</sup>

Given the insider knows the expected growth rate  $\hat{m}_{nT}$  at  $t = nT - 1$ , there is no uncertainty of the underlying fundamental to her since then. Thus,  $A\text{-SDF}_{t,nT}^{\text{insider}} \equiv 1$  under the insider's information set for all  $t \in [nT - 1, nT]$ . In other words, the insider is "risk-neutral" toward the news contained in announcements due to her perfect knowledge of the

<sup>22</sup>See the proof in Lemma 5.

<sup>23</sup>It can be extended to a general smooth distribution as shown in the proof of section 5 in the appendix.

<sup>24</sup>One example of  $A(\hat{m}_t, t) \approx e^{\frac{\phi-1}{am+e^{-\rho}} \hat{m}_t + N(t)}$  is through the stock which has the claim to the following dividend process:

$$\frac{dD_t}{D_t} = [\bar{m} + \phi(m_t - \bar{m})] dt + \phi \sigma_C d\tilde{B}_{C,t}, \quad (11)$$

where we allow the leverage parameter  $\phi \geq 1$  so that dividends are more risky than consumption, as in Bansal and Yaron (2004). In addition, the shock  $dB_{D,t}$  is independent of  $dB_{C,t}$  and  $dB_{m,t}$ . The proof is in the appendix.

underlying information.<sup>25</sup> The insider maximizes the expectation of her terminal profit:

$$\begin{aligned}
& J(nT-1, P_{nT-1}, A(\hat{m}_{nT}, nT)) \\
&= \max_{X_t} \mathbb{E} \left[ \int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P_t) dX_t \mid \mathcal{F}_{nT-1}^Y, A(\hat{m}_{nT}, nT) \right] \\
&= \max_{\theta_t \in \mathcal{A}} \mathbb{E} \left[ \int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P_t) \theta_t dt \mid \mathcal{F}_{nT-1}^Y, A(\hat{m}_{nT}, nT) \right]. \tag{12}
\end{aligned}$$

In addition to the entire past history of aggregate order flow  $Y$ , the insider knows the actual value of the stock  $A(\hat{m}_{nT}, nT)$ , and, of course, her own trading. Following Back (1992), I assume that the insider chooses an absolutely continuous trading rule  $dX_t = \theta_t dt$  that belongs to an admissible set  $\mathcal{A} = \left\{ \theta \text{ s.t. } \mathbb{E} \left[ \int_{nT-1}^{nT} \theta_s^2 ds \right] < \infty \right\}$ . Therefore, the dynamics of aggregate order flow  $Y$  is the sum of the insider's demand the liquidity traders' demand:

$$dY_t = \theta_t dt + dZ_t.$$

## 4.2 The equilibrium

**Definition 1.** *An equilibrium is a price process and an admissible trading strategy,  $(P_t, \theta_t)$ , that satisfy the market makers' rationality condition (10) while solving the insider's optimality condition (12).*

The introduction of the risk-compensated market makers leads to the following main difference comparing to the standard Kyle model and related extensions in the literature.<sup>26</sup> Instead of only estimating the (discounted) value of the risky asset, the market makers also update the A-SDF simultaneously before announcements based on the observed cumulative order flow. In the standard Kyle model, the risk-neutral market makers only estimate the value of  $A(\hat{m}_{nT}, nT)$  and set the price

$$P_t^{\text{Kyle}} = \mathbb{E} \left[ A(\hat{m}_{nT}, nT) \mid \mathcal{F}_t^Y \right], \tag{13}$$

<sup>25</sup>This can be shown directly through equation (9) where I take the expectation under the insider's information set at  $t \in [nT-1, nT]$ .

<sup>26</sup>See Kyle (1985), Back (1992), Collin-Dufresne and Fos (2016), Back, Crotty, and Li (2018), etc.

In equilibrium,  $P_t^{\text{Kyle}}$  must be a martingale under market makers' information set. While when market makers are compensated for the risk of assets' fundamentals, the pricing rule from (10) can be rewritten as

$$P_t = \frac{\mathbb{E} [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y]}{\mathbb{E} [H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y]} \equiv \frac{V_t}{\Lambda_t}, \quad (14)$$

where  $V_t$  and  $\Lambda_t$  are the market makers' estimation of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  and  $H(\hat{m}_{nT}, nT)$ , respectively. In equilibrium, both  $V_t$  and  $\Lambda_t$  are martingales under market makers' information set. However, the pricing rule probably not. Later I will show  $P_t$  is a *submartingale* with respect to the market makers' information if and only they are compensated for the risk of assets' fundamentals. The intuition is that the uncertainty of underlying fundamental is resolved gradually after observing aggregate order flow, which is associated with the realization of the premium when the market makers are compensated for risk-taking.

To solve for an equilibrium, I proceed in a few steps. First, in Lemma 2, conditional on a conjectured insider's trading strategy, I derive the stock price dynamics consistent with the market makers' filtering. Then, given the assumed dynamics of the equilibrium price, I solve the insider's optimal trading strategy that is captured in Lemma 3 and Lemma 4. Finally, I show that the conjectured rule by the market makers is indeed consistent with the insider's optimal choice, as stated in Theorem 1.

**Lemma 2.**  $\forall t \in [nT - 1, nT]$ , suppose the insider adopts the following trading strategy

$$\theta_t = \frac{\log [A(\hat{m}_{nT}, nT)] - \mu_P + \gamma^A \beta \Delta Q}{(nT - t) \lambda} - \frac{Y_t}{nT - t} \quad (15)$$

where  $\mu_P = \beta \hat{m}_{nT-1} + N(nT)$ ,  $\sigma_v^2 = \beta^2 \Delta Q$ , and  $\lambda = \frac{\sigma_v}{\sigma_z}$ . Then the market makers' estimations given by equation (14) satisfy the stochastic differential equations

$$\frac{dV_t}{V_t} = \frac{\beta - \gamma^A}{\beta} \lambda [dY_t - \hat{\theta}_t dt] \equiv \frac{\beta - \gamma^A}{\beta} \lambda d\hat{Y}_t, \quad (16)$$

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{-\gamma^A}{\beta} \lambda [dY_t - \hat{\theta}_t dt] \equiv \frac{-\gamma^A}{\beta} \lambda d\hat{Y}_t. \quad (17)$$

The expected insider's order rate under the market makers' filtration  $\mathcal{F}_t^Y$  is

$$\hat{\theta}_t \equiv \mathbb{E} \left[ \theta_t | \mathcal{F}_t^Y \right] = \frac{\gamma^A \beta \Delta Q}{\lambda}, \quad (18)$$

and the adjusted order flow  $\hat{Y}_t$

$$\hat{Y}_t \equiv Y_t - \int_{nT-1}^t \hat{\theta}_s ds = Y_t - \frac{\gamma^A \beta \Delta Q}{\lambda} [t - (nT - 1)], \quad (19)$$

is a Brownian Motion with instant variance  $\sigma_z^2$  with respect to the market makers' filtration  $\mathcal{F}_t^Y$ .

Further, the market makers' pricing rule in equation (14) is a function of  $(t, \hat{Y}_t)$  that follows

$$\frac{dP(t, \hat{Y}_t)}{P(t, \hat{Y}_t)} = \lambda d\hat{Y}_t + \gamma^A \beta \Delta Q dt, \quad \text{with } P_{nT-1} = e^{\mu_P - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 + \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2}. \quad (20)$$

When market makers are risk neutral, aggregate order flow  $Y_t$  at equilibrium is a martingale under the market makers' information set, as shown by Back (1992). In other words, the market makers are to set the pricing rule such that the expected order rate from the insider is zero. While market makers are risk averse to the underlying fundamental (i.e.,  $\gamma^A > 0$ ), equation (18) indicates that the expected insider's order rate under the market makers' filtration  $\mathcal{F}_t^Y$  is strictly positive. Here is the intuition behind this result. The information from aggregate order flow resolves market makers' uncertainty before FOMC announcements. Since they are compensated for risk-taking, the equity premium realized gradually during this period. This leads to an average upward drift in market prices. Therefore, market makers would expect an average positive trading volume from the insider to chase that premium. Besides, the insider has to consider this additional price impact from uncertainty resolution when they trade, which is unique in this model.

The above analysis implies aggregate order flow  $Y_t$  at equilibrium is no longer a martingale under  $\mathcal{F}_t^Y$  when  $\gamma^A > 0$ . More importantly, since the average positive order flow from the insider is expected, market makers would update their estimates from the adjusted order flow  $\hat{Y}_t$  instead of aggregate order flow  $Y_t$ . Thus, I suppose that there exists an equilibrium with two state variables: time  $t$  and the adjusted order flow  $\hat{Y}_t$ . Then given the

make makers' pricing rule,  $P(t) = P(t, \hat{Y}_t)$ , the insider chooses the order rate to maximize her trading profit. That is,

$$J(t, y, A(\hat{m}_{nT}, nT)) = \max_{\theta_t \in \mathcal{A}} \mathbb{E} \left[ \int_t^{nT} (A(\hat{m}_{nT}, nT) - P(s, \hat{Y}_s)) \theta_s ds \mid \hat{Y}_t = y, A(\hat{m}_{nT}, nT) \right]$$

subject to

$$d\hat{Y}_t = [\theta_t - \hat{\theta}_t] dt + dZ_t, \quad \text{where } \hat{\theta}_t \equiv \mathbb{E} [\theta_t \mid \mathcal{F}_t^Y]. \quad (21)$$

The principle of optimality implies the following Bellman equation

$$\max_{\theta_t \in \mathcal{A}} \left\{ (A(\hat{m}_{nT}, nT) - P(t, y)) \theta_t + J_t + J_y [\theta_t - \hat{\theta}_t] + \frac{1}{2} \sigma_z^2 J_{yy} \right\} = 0, \quad (22)$$

where the subscripts denote the derivatives. The necessary conditions for having an optimal solution to the Bellman equation (22) are

$$J_y(t, y, A(\hat{m}_{nT}, nT)) = P(t, y) - A(\hat{m}_{nT}, nT), \quad (23)$$

$$J_t + \frac{1}{2} \sigma_z^2 J_{yy} - \hat{\theta}_t J_y = 0. \quad (24)$$

These necessary conditions lead to the following results.

**Lemma 3.** *Suppose the expected order rate  $\hat{\theta}(t) = \Theta(t, \hat{Y}_t)$ , where  $\hat{Y}_t$  is the adjusted order at  $t$ . Let  $\omega_t = y$  and suppose that the stochastic differential equation*

$$d\omega_s = dZ_s - \Theta(s, \omega_s) ds, \quad \forall nT \geq s \geq t \geq nT - 1$$

*has a unique solution, where  $Z_s$  is a Brownian motion with instant variance  $\sigma_z^2$ .<sup>27</sup> If there exists a strictly monotone function  $g(\cdot)$  such that the pricing rule is*

$$P(t, y) = \mathbb{E} [g(\omega_{nT}) \mid \omega_t = y], \quad (25)$$

*then*

$$J(t, y, A(\hat{m}_{nT}, nT)) = \mathbb{E} [j(\omega_{nT}, A(\hat{m}_{nT}, nT)) \mid \omega_t = y], \quad (26)$$

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<sup>27</sup>Note that  $\omega_t$  is the adjusted order when the insider does not submit orders.



is a smooth solution to the Bellman equations (23) and (24), where

$$j(y, A(\hat{m}_{nT}, nT)) = \int_y^{g^{-1}(A(\hat{m}_{nT}, nT))} [A(\hat{m}_{nT}, nT) - g(x)] dx \geq 0, \quad \forall (y, A(\hat{m}_{nT}, nT)).$$

**Lemma 4.** Any continuous trading strategy that makes  $\lim_{t \rightarrow nT} P(t, \hat{Y}(t)) = A(\hat{m}_{nT}, nT)$  is optimal, where  $P(t, y)$  is as defined by equation (25).

Having established these results, I can now proceed to characterize the equilibrium price and the insider's optimal strategy. The equilibrium I obtain, which constitutes the main results of this paper, is summarized in the following theorem.

**Theorem 1.**  $\forall t \in [nT - 1, nT]$ , there exists an equilibrium where the price process  $P_t$  and optimal strategy of the insider  $\theta_t$  have dynamics,

$$\frac{dP(t, \hat{Y}_t)}{P(t, \hat{Y}_t)} = \lambda d\hat{Y}_t + \gamma^A \beta \Delta Q dt, \quad (27)$$

$$\theta(t, \hat{Y}_t) = \frac{\log[A(\hat{m}_{nT}, nT)] - \mu_P}{(nT - t)\lambda} - \frac{\hat{Y}_t}{nT - t} + \frac{\gamma^A \beta \Delta Q}{\lambda}, \quad (28)$$

where  $\hat{Y}_t$ ,  $P_{nT-1}$ ,  $\mu_P$ ,  $\sigma_v$ , and  $\lambda$  are defined in Lemma 2. The expected insider's order rate under  $\mathcal{F}_t^Y$  is defined in equation (18).

The maximized expected profit of the insider is

$$J(t, P(t, \hat{Y}_t), A(\hat{m}_{nT}, nT)) = \frac{1}{2} \sigma_z \sigma_v (nT - t) A(\hat{m}_{nT}, nT) + \frac{P(t, \hat{Y}_t) - A(\hat{m}_{nT}, nT) + A(\hat{m}_{nT}, nT) [\log A(\hat{m}_{nT}, nT) - \log P(t, \hat{Y}_t)]}{\lambda}. \quad (29)$$

With respect to the insider's filtration,  $P(t, \hat{Y}_t)$  converges almost surely to  $A(\hat{m}_{nT}, nT)$  at time  $t = nT$ . When the market makers are risk-compensated, with respect to the market makers' filtration, both the pricing rule  $P(t, \hat{Y}_t)$  and the price-response coefficient  $P_{\hat{Y}}(t, \hat{Y})$  are submartingales with a constant growth rate  $\gamma^A \beta \Delta Q$ .

Further,  $\forall t \in [nT - 1, nT]$ , the expected cumulative pre-FOMC announcement drift is

$$\log \mathbb{E} \left[ \frac{P_t}{P_{nT-1}} \middle| \mathcal{F}_{nT-1}^Y \right] = \gamma^A \beta \Delta Q (t - (nT - 1)). \quad (30)$$

This implies there is a strictly positive pre-FOMC announcement drift if and only if the market makers are compensated for the risk of assets' fundamentals, i.e.,  $\gamma^A > 0$ .

I now comment on several implications of the theorem. First, the equilibrium price is a submartingale, expected to increase over time. This contrasts my framework from much of the literature. They find the price dynamics is a martingale under the risk-neutral market makers since they are indifferent, to resolve the uncertainty now or in the future. While when market makers are risk compensated, the resolution of uncertainty is associated with the realizations of the premium. The positive expected pre-FOMC announcement premium is cumulated at a constant rate  $\gamma^A \beta \Delta Q$ , which is the negative covariance between the innovation to the A-SDF and the asset value. Intuitively, the pre-announcement drift would be larger: (1) when market makers are more risk-averse to the underlying fundamental; (2) the asset value has a larger exposure to the FOMC news; (3) more transparent FOMC announcements which reduce more uncertainty. In addition, the equilibrium price converges to the value  $A(\hat{m}_{nT}, nT)$ , known ex ante only to the insider, at FOMC announcements. This guarantees all of the private information is eventually incorporated into the price and generalizes the result proved in Back (1992) under the risk-neutral market makers.

Second, I find when the market makers are risk averse to the underlying fundamental (i.e.,  $\gamma^A > 0$ ), the expected insider's order rate under the market makers' filtration  $\mathcal{F}_t^Y$  follows

$$\mathbb{E} \left[ \theta_t \middle| \mathcal{F}_t^Y \right] = \frac{\gamma^A \beta \Delta Q}{\lambda} = \gamma^A \sqrt{\Delta Q} \sigma_z, \quad (31)$$

which is strictly positive. This is very different from the Kyle model, where the expected insider's order rate is always zero. Here is the intuition. Due to the average upward drift in market prices, the market makers rationally anticipate that the insider would trade positively on average to chase that premium. The insider also has to consider the additional price impact from uncertainty resolution when she trades, which is unique in this model.

The equilibrium expected insider's order rate is determined by the ratio of the expected pre-FOMC announcement premium per unit of time ( $\gamma^A \beta \Delta Q$ ) to Kyle's lambda ( $\lambda$ ). Therefore, the abnormal order imbalances are on average positive when there is private information before FOMC announcements. In addition, the insider would on average trade more aggressively when market makers are more risk-averse to the uncertainty or FOMC announcements are more transparent, which is caused by the higher realized equity premium per unit of time. In the meantime, when noise traders are more active, the insider on average trades more due to the smaller price impact, which has been largely missed in the Kyle-type models.<sup>28</sup>

Third, the price impact  $P_{\hat{Y}}(t, \hat{Y})$  is also a submartingale, which grows at the same rate as the equilibrium price. The risk-averse market makers benefit from uncertainty resolution out of observing aggregate order flow. Therefore, to entice the insider to trade and release information early, the market makers have incentives to set the price impact that increases on average. Collin-Dufresne and Fos (2016) is one of the few papers that achieve the same result through a different channel that comes from the insider's potential benefit to wait for better liquidity with stochastic noise trading volatility.<sup>29</sup>

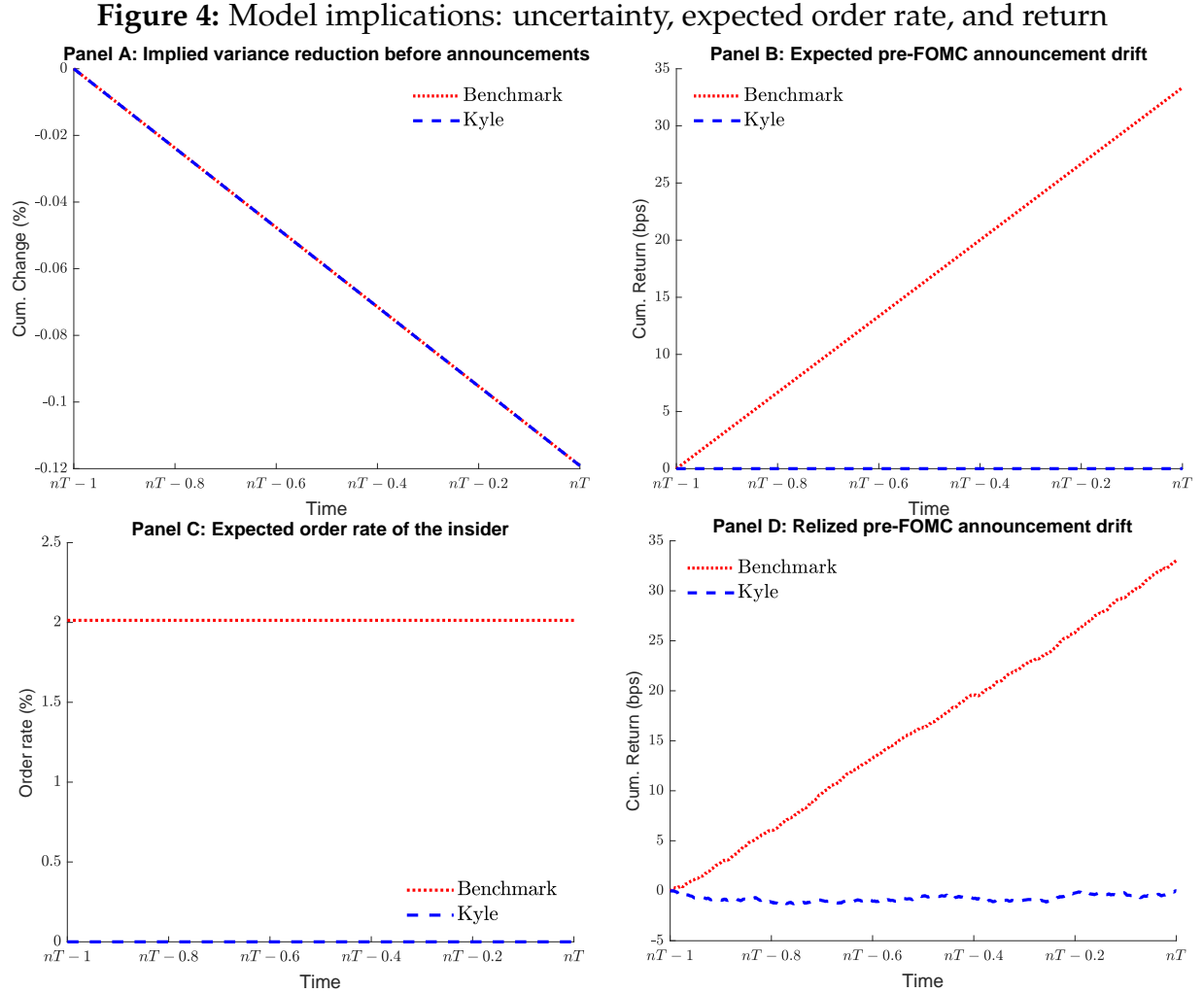
Fourth, when the market makers converge to be risk-neutral, the limit of the equilibrium is well defined and converges to the traditional Kyle model (more precisely, converges to Back (1992)). When  $\gamma^A$  converges to zero, the expected insider's order rate  $\hat{\theta}_t$  converges to zero, which implies aggregate order flow  $Y_t$  converges to a martingale. The pricing rule  $P(t, \hat{Y}_t)$  and the price-response coefficient  $P_{\hat{Y}}(t, \hat{Y})$  also converge to martingales. This implies the expected pre-announcement drift converges to a flat line, as in Back (1992). The convergence result demonstrates that there is a strictly positive pre-FOMC announcement drift if and only if the market makers are compensated for the risk of assets' fundamentals, i.e.,  $\gamma^A > 0$ , as proved in Theorem 1.

<sup>28</sup>The only exception is Collin-Dufresne and Fos (2016) that derive the same result by assuming noise trading volatility follows a general stochastic process.

<sup>29</sup>The price impact is constant in Kyle (1985). In extensions of that model Back (1992), Back and Pedersen (1998), Baruch (2002), Back and Baruch (2004), Caldentey and Stacchetti (2010), price impact is either a martingale or a supermartingale. Collin-Dufresne and Fos (2016) points that Foster and Viswanathan (1996) and Back, Cao, and Willard (2000) may also generate an increase in the deterministic price impact, at least near the end of the trading horizon, because of competition among multiple informed traders.

### 4.3 Properties of equilibrium

Having characterized the equilibrium, in this section, I study the equilibrium properties and map the model to asset market fluctuations before FOMC announcements.



This figure shows the model implications for the cases with the risk-compensated market makers (Benchmark) and the risk-neutral market makers (Kyle) as a function of time, respectively. Panel A plots the implied variance change before announcements. Panel B plots the expected pre-FOMC announcement excess return. Panel C plots the expected insider's order rate under the market makers' filtration  $\mathcal{F}_t^Y$ . Panel D plots the average realized pre-FOMC announcement return, which is computed from 10,000 parallel samples. The parameters are reported in Table 1.

The following proposition captures the uncertainty reduction prior to announcements in the equilibrium from Theorem 1.

**Proposition 1.** *With respect to the market makers' filtration  $\mathcal{F}_t^Y$ ,  $\forall t \in [nT - 1, nT]$ , the uncertainty reduction at time  $t$  comparing to  $nT - 1$  follows*

$$\text{Var} \left[ \log P_{nT} | \mathcal{F}_t^Y \right] - \text{Var} \left[ \log P_{nT} | \mathcal{F}_{nT-1}^Y \right] = -\beta^2 \Delta Q [t - (nT - 1)]. \quad (32)$$

*Thus, prior to announcements, the uncertainty reduces at a constant rate  $\beta^2 \Delta Q$  per unit of time.*

So far,  $\forall t \in [nT - 1, nT]$ , I explicitly characterize the expected pre-FOMC announcement drift and the implied variance reduction in equations (30) and (32), respectively. I take the macroeconomic parameters from Ai and Bansal (2018) and calibrate the risk aversion and the transparency of announcements to match the level of cumulative return and uncertainty reduction upon announcements.<sup>30</sup> The parameters are reported in Table 1. I call this case as the benchmark where the market makers are risk-compensated. For comparison, I study another case where I keep other parameters the same and assume the market makers are risk-neutral, which is equivalent to the original Kyle model with a log-normal distribution of the asset value (see Back (1992)).

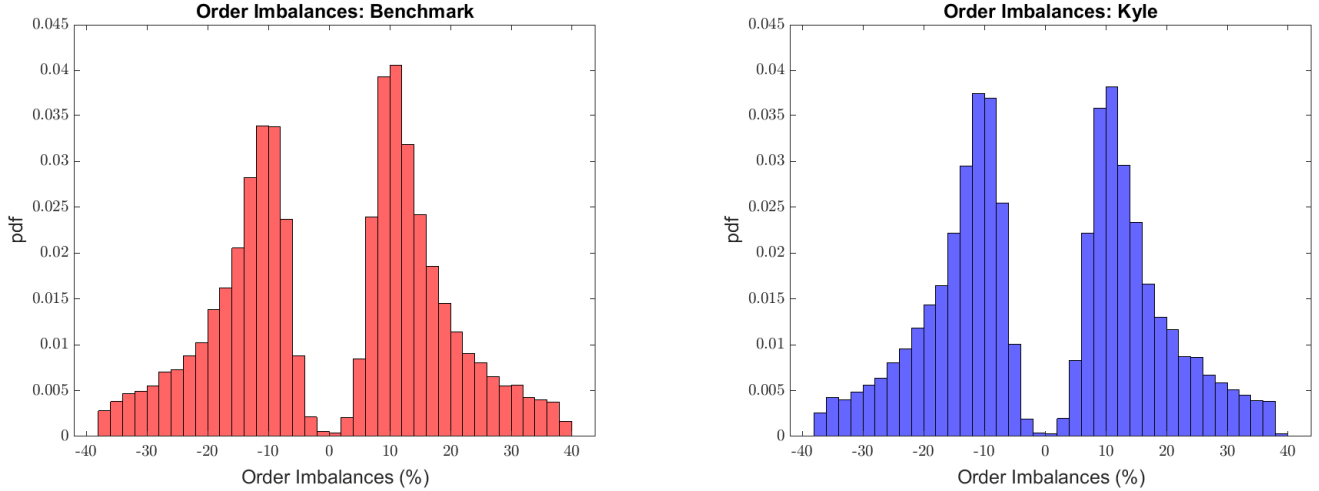
Figure 4 depicts the model implications for the benchmark case (the dotted red line) and the Kyle case (the dashed blue line) as a function of time, respectively. Panel A plots the implicated variance changes before announcements defined in equation (32), which are the same for both cases.<sup>31</sup> This is because the implicated variance reduction only depends on the risk exposure  $\beta$ , which is not a function of  $\gamma^A$ . However, the expected pre-FOMC announcement excess returns are very different, as shown in Panel B. When the market makers are risk-compensated, the expected pre-announcement drift has a constant positive rate as captured in equation (30). The expected pre-announcement drift is literally zero with the risk-neutral market makers. Panel C compares the expected insider's order rate under the market makers' information set, which is strictly positive with the risk-averse market makers as captured in equation (31). Panel D plots the average realized pre-FOMC announcement excess returns that are computed from parallel simulations, respectively. This is consistent with my previous discussion that there is a strictly positive pre-FOMC announcement drift

<sup>30</sup>See the full model calibration in section 5.3.

<sup>31</sup>For convenience, I match the level of the cumulative change of the  $VIX^2$  instead of the  $VIX$  displayed in Figure 1.

if and only if the market makers are compensated for the risk of assets' fundamentals.

**Figure 5: Model implications: order imbalances**



This figure shows the distribution of the order imbalances for the cases with the risk-compensated market makers (Benchmark) and the risk-neutral market makers (Kyle). I simulate 50,000 times in the frequency of one minute and winsorize the order imbalances at the 10th and 90th percentiles to mitigate the effect of outliers. The parameters are reported in Table 1.

I follow the empirical procedure—the tick rule to calculate the order imbalances in the model. The aggregate buyer-initiated ( $OI_B$ ) and seller-initiated ( $OI_S$ ) dollar trading volumes are defined as

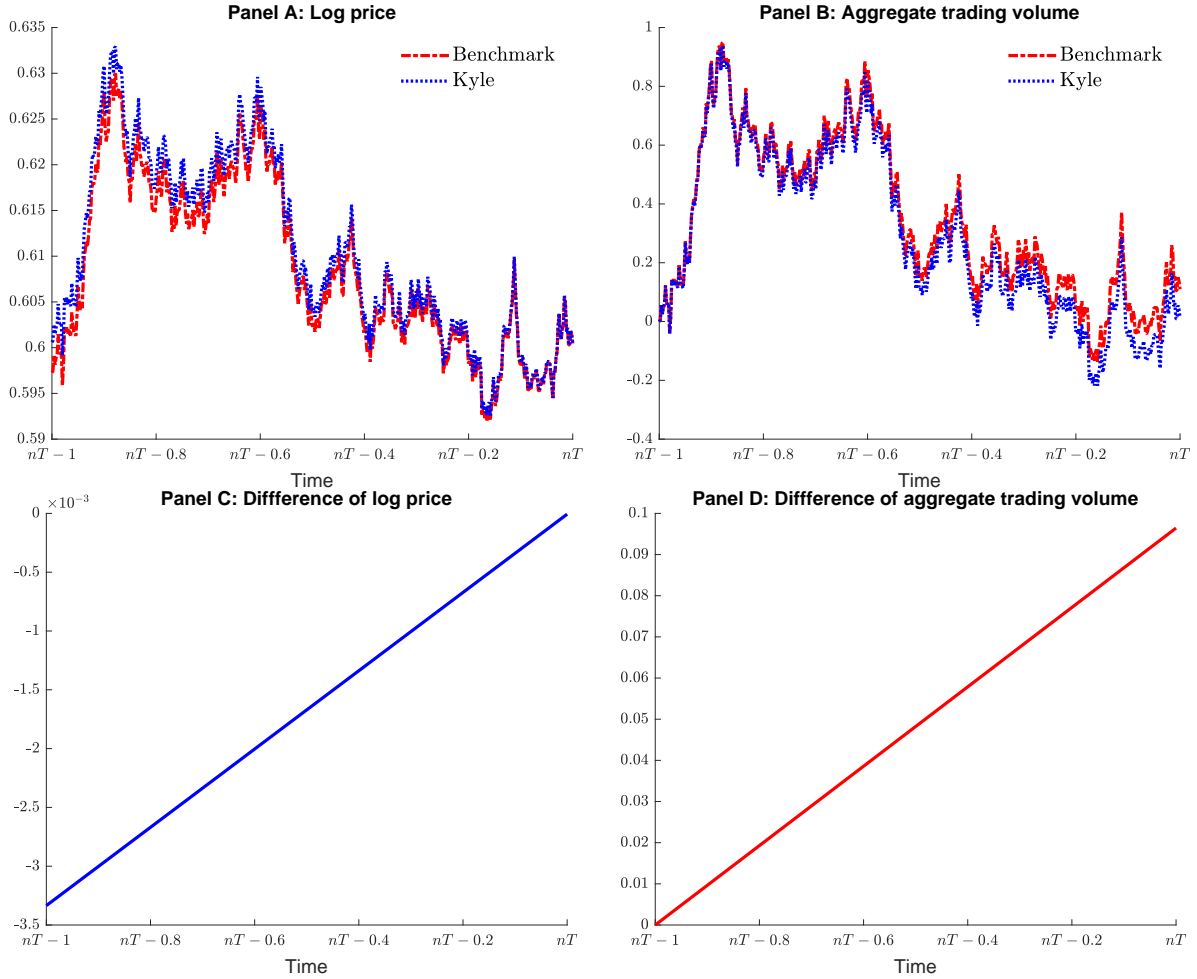
$$OI_B = \int_{nT-1}^{nT} p_t \theta_t 1_{\{p_t > p_{t-1}\}} dt, \quad OI_S = \int_{nT-1}^{nT} p_t \theta_t 1_{\{p_t < p_{t-1}\}} dt, \quad (33)$$

respectively. Thus, the order imbalances are  $\frac{OI_B - OI_S}{OI_B + OI_S}$ . Figure 5 shows the distribution of the order imbalances for the benchmark and the Kyle model. The average of the order imbalances in the benchmark is 0.85%, which is statistically significant at the 1% level. The significant positive order imbalances come from the positive excess return before FOMC announcements. While in the Kyle model, the average is not significantly different from zero that is not consistent with data.

So far, I compare the proprieties of average variables in the benchmark to the Kyle model. To have a better understanding of the equilibrium in Theorem 1, the following proposition

emphasizes the differences between the two cases under any realized path of  $Z_t$ .

**Figure 6: Model implications: under one realized path of  $Z_t$**



This figure shows the model implications for the cases with the risk-compensated market makers (Benchmark) and the risk-neutral market makers (Kyle) as a function of time, under one realized path of  $Z_t$ . Panel A plots the dynamics of the log price. Panel B plots the aggregate trading flow. Panel C plots the difference of log price  $\log P_t - \log P_t^{\text{Kyle}}$ . Panel D plots the difference of the aggregate trading flow  $Y_t - Y_t^{\text{Kyle}}$ . The parameters are reported in Table 1.

**Proposition 2.** Under the same realization path of  $Z_t$ , the difference between the price  $P_t$  in the benchmark ( $\gamma^A > 0$ ) and the price  $P_t^{\text{Kyle}}$  in Kyle ( $\gamma_{\text{Kyle}}^A = 0$ ) increases at a constant rate:

$$\log P_t - \log P_t^{\text{Kyle}} = \gamma^A \beta \Delta Q [t - nT], \quad \forall t \in [nT - 1, nT],$$

which converges to zero upon announcements. Meanwhile, the difference between the aggregate cumulative trading flow  $Y_t$  and  $Y_t^{\text{Kyle}}$  increases at a constant rate:

$$Y_t - Y_t^{\text{Kyle}} = \frac{\gamma^A \beta \Delta Q}{\lambda} [t - (nT - 1)], \quad \forall t \in [nT - 1, nT].$$

Figure 6 plots the log price dynamics and aggregate trading volume for the benchmark case (the dotted red line) and the Kyle case (the dashed blue line) under one realized path of  $Z_t$ .<sup>32</sup> Panel A and B plot the dynamics of the log price and the aggregate trading flow as a function of time, repetitively. The two cases show similar patterns of fluctuations, which are consistent with the linear differences in Panel C and D. When the market makers are risk-averse to the underlying fundamental, the initial price  $P_{nT-1}$  has to be lower to attract them to hold the assets. The difference of the price converges to zero since all of the private information is eventually incorporated into the price for both cases, i.e.,  $P_{nT} = P_{nT}^{\text{Kyle}} = A(\hat{m}_{nT}, nT)$  almost surely. The insider in the benchmark trades more aggressively to chase the realized premium, which results in the larger aggregate trading volume over time.

## 5 Full model: the insider may not be informed

Figure 2 indicates that not all FOMC announcements are the same—some are not associated with uncertainty reduction prior to announcements. Motivated by this fact, I extend the above model to the case that the insider may or may not be informed of the signal  $s_n$  before announcements.<sup>33</sup> In the meantime, the market makers are not sure whether the insider observes the signal or not. The market makers share a common belief that such an event, in which the insider observes this information earlier than the public, occurs with a probability  $\pi_{nT-1} \in (0, 1)$  at time 0. Therefore, in addition to the discounted value of the risky asset and the A-SDF, the market makers also have to update their estimate of the probability that the insider has private information of FOMC announcements.

<sup>32</sup>Note that although Figure 6 plots only one realized path of  $Z_t$ , it represents the typical situation in the model.

<sup>33</sup>This extension is based on Li (2013), which extends Back (1992) to study the insider trading with uncertain informed trading. He keeps the assumption of the risk-neutral market makers.



## 5.1 Model setting

Let  $X_{\delta,t}$  denote the net orders from the insider trader. Then the total cumulative order flow  $Y_t$  can be expressed as

$$Y_t = X_{\delta,t} + Z_t,$$

where  $\delta$  is an indicator function, which is equal to 1 if the insider has information and is equal to 0 otherwise. By observing this order flow, the market makers update their estimates about the *probability* that the insider possesses private information and the *value* of the risky security. Let  $\mathcal{F}_{0,t} = \mathcal{F}_t^Y \times \{\delta = 0\}$  under the hypothesis  $\delta = 0$  and  $\mathcal{F}_{1,t} = \mathcal{F}_t^Y \times \{\delta = 1\}$  under the hypothesis  $\delta = 1$ . I let  $\pi(t) = \mathbb{E}[\delta | \mathcal{F}_t^Y]$  be the estimate of the probability that the insider has private information at time  $t$ .

If the insider does not have any private information ( $\delta = 0$ ), she has no information other than what the market makers have. Therefore,  $\forall t \in [nT - 1, nT]$ , the best estimate of the security's value is

$$\begin{aligned} \bar{v}^* &\equiv \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t} \right], \\ &= \frac{\mathbb{E}[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}]}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{0,t}]} \equiv \frac{\bar{V}}{\bar{\Lambda}}, \end{aligned} \quad (34)$$

where I define  $\bar{V}$  and  $\bar{\Lambda}$  as the estimate of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  and  $A(\hat{m}_{nT}, nT)$  under the case that the insider is not informed, respectively.

If the insider has private information ( $\delta = 1$ ), the value estimate of the risky security at time  $t$  conditional on  $\delta = 1$  is

$$\begin{aligned} v^*(t) &\equiv \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t} \right], \\ &= \frac{\mathbb{E}[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}]}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}]} \equiv \frac{V(t)}{\Lambda(t)}, \end{aligned} \quad (35)$$

where I define  $V(t)$  and  $\Lambda(t)$  as the estimate of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  and  $A(\hat{m}_{nT}, nT)$  under the case that the insider is informed, respectively.

With the uncertainty of  $\delta$ , the market makers estimate the discounted value under the information structure  $\mathcal{F}_{1,t}$  and estimate the probability that the insider has observed private information under the information structure  $\mathcal{F}_t^Y$ . Given these two estimates, the market makers set the price that follows

$$\begin{aligned}
P(t) &= \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y]} A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right], \\
&= \frac{\mathbb{E}[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y]}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y]}, \\
&= \frac{\pi(t) V(t) + (1 - \pi(t)) \bar{V}}{\pi(t) \Lambda(t) + (1 - \pi(t)) \bar{\Lambda}}.
\end{aligned} \tag{36}$$

Note that when the market makers know the insider is always informed ( $\pi_{nT-1} = 1$ ), the market makers set the price as

$$P(t) = \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t} \right],$$

which goes back to the benchmark model in section 4.

I impose the following restriction on the market makers' value estimates  $V(t)$  and  $\Lambda(t)$  conditional on  $\delta = 1$  defined in equation (35):

$$\mathbb{E} \left[ \int_{nT-1}^{nT} V^2(s) ds \right] < \infty; \quad \mathbb{E} \left[ \int_{nT-1}^{nT} \Lambda^2(s) ds \right] < \infty.$$

This restriction implies that the pricing rule defined by equation (36) satisfies

$$\mathbb{E} \left[ \int_{nT-1}^{nT} P^2(s) ds \right] < \infty,$$

which is sufficient to rule out the so-called doubling strategy that the insider could use.<sup>34</sup>

## 5.2 The equilibrium

**Definition 2.** *An equilibrium is an quadruple  $(X_0, X_1, P, \Pi)$  such that*

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<sup>34</sup>See Back (1992) and Li (2003) for more details.

1. both  $X_0$  and  $X_1$  are the optimal trading strategies of the insider when she has not or has observed private information, respectively, given  $P(t)$  and  $\Pi$ ;
2.  $P(t) = \frac{\Pi(t)V(t) + (1-\Pi(t))\bar{V}}{\Pi(t)\Lambda(t) + (1-\Pi(t))\bar{\Lambda}}$  is the stock price at time  $t$ , where  $V(t)$  and  $\Lambda(t)$  are the market makers' value estimates of the risky security and SDF conditional on  $\delta = 1$ , and  $\Pi(t) = \pi(t)$ , is the market makers' probability estimates that the insider has private information, given the insider trader's trading strategies  $X_0$  and  $X_1$ .

As the benchmark in section 4, I assume that the insider chooses an absolutely continuous trading rule

$$dX_{1,t} = \theta(t, \tilde{V}) dt.$$

$\theta(t, \tilde{V})$  belongs to an admissible set  $\mathcal{A} = \left\{ \theta \text{ s.t. } \mathbb{E} \left[ \int_{nT-1}^{nT} \theta^2(t, \tilde{V}) ds \right] < \infty \right\}$ , where  $\tilde{V}$  is the insider's perfect knowledge of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$ .<sup>35</sup> When the insider has no information other than what the market makers have, her order rate becomes  $\theta(t, \bar{V})$ . Given this trading strategy, the cumulative flow is

$$Y_t = \int_{nT-1}^t \theta(s, V) ds + Z_t. \quad (37)$$

From the market makers' point of view, the cumulative order flow has two possible interpretations because they don't know how much the liquidity traders trade. One is

$$dY_t = \theta(t, \tilde{V}) dt + dZ_t, \quad (38)$$

if the insider is informed, and the other is

$$dY_t = \theta(t, \bar{V}) dt + dZ_t, \quad (39)$$

if the insider is not informed.

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<sup>35</sup>Note that the order rate of the insider should also depend on the market makers' pricing rule or some other state variable(s). I omit such state variables in the expression of the order rate because what these variables are is not clear yet. Besides, both of the estimation of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  and  $A(\hat{m}_{nT}, nT)$  rely on the belief updates of  $\hat{m}_{nT}$ . In other words, inferring one of  $V(t)$  and  $\Lambda(t)$  is enough. Therefore, I write  $\theta(t, \tilde{V})$  as a function of  $t$  and  $V(t)$ .

The following assumption imposes that when the insider is not informed, she will not take a dramatically different trading strategy. Otherwise, her trading behavior may immediately reveal that she does not have private information for a specific FOMC announcement.<sup>36</sup>

**Assumption 1.** *When the insider is not bettered informed, she maximizes the following terminal profit under her best estimation of the asset value,*

$$\begin{aligned} & \int_{nT-1}^{nT} \left( \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1}]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1} \right] - P_s \right) \theta_s ds \\ &= \int_{nT-1}^{nT} (\bar{v}^* - P_s) \theta_s ds. \end{aligned}$$

Given the observation of the cumulative order flow, the market makers update the probability that the insider has private information, the A-SDF, as well as the discounted value of the security conditional on the insider is informed. These estimates are done by solving a nonlinear filtering problem. The equilibrium is summarized in the following theorem (proved in the Appendix).

**Theorem 2.** *Under Assumption 1,  $\forall t \in [nT - 1, nT]$ , there exists an equilibrium  $(X_0, X_1, P, \Pi)$  that follows:*

(1). *The market makers' probability estimate  $\Pi(t, y)$  is*

$$\Pi(t, y) = \frac{\pi_{nT-1} \exp \left( \frac{1}{2\sigma_z^2} \frac{[y-\bar{y}]^2}{nT-t} + \frac{1}{2} \log(nT-t) - \frac{\bar{y}^2}{2\sigma_z^2} \right)}{1 - \pi_{nT-1} + \pi_{nT-1} \exp \left( \frac{1}{2\sigma_z^2} \frac{(y-\bar{y})^2}{nT-t} + \frac{1}{2} \log(nT-t) - \frac{\bar{y}^2}{2\sigma_z^2} \right)}, \quad (40)$$

where  $y$  represents the adjusted order flow  $\hat{Y}_{1,t}$  (defined later) and  $\bar{y} = \frac{\beta - \gamma^A}{\beta} \sigma_v^2$ ;

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<sup>36</sup>It is possible that the insider observes a private signal, indicating that the terminal value is  $\bar{v}^*$ . Under this case, since the insider has no information advantage comparing to market makers' prior, I interpret it as the insider having no private information.

(2). The pricing rule  $P(t, y)$  has dynamics

$$P(t, y) = P_{nT-1} \frac{\Pi(t, y) e^{\frac{\beta-\gamma^A}{\beta} \lambda y - \frac{1}{2} \left(\frac{\beta-\gamma^A}{\beta}\right)^2 \sigma_v^2 (t-(nT-1))} + 1 - \Pi(t, y)}{\Pi(t, y) e^{-\frac{\gamma^A}{\beta} \lambda y - \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (t-(nT-1))} + 1 - \Pi(t, y)}, \quad (41)$$

where  $P_{nT-1}$ ,  $\sigma_v$ , and  $\lambda$  are defined in Lemma 2;

(3). The insider's trading strategy  $X_\delta(t, y)$  satisfies

$$X_1(t, y) = \int_{nT-1}^t \theta(s, y; H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)) ds, \quad X_0(t, y) = \int_{nT-1}^t \theta(s, y; \bar{V}) ds. \quad (42)$$

The insider's order rate for any  $\tilde{V}$  is

$$\theta(t, y; \tilde{V}) = \bar{\theta}(t, y) + \frac{(\log \tilde{V} - \mu_V) / \left(\frac{\beta-\gamma^A}{\beta} \lambda\right) - \bar{y} - \Pi(t, y) [y - \bar{y}]}{nT - t},$$

where  $\mu_V = (\beta - \gamma^A) \hat{m}_{nT-1} + \mathcal{H}(nT) + N(nT)$  is the mean of  $\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]$ .

The expected order rate of the insider  $\bar{\theta}(t, y)$  under the market makers' filtration  $\mathcal{F}_t^Y$  satisfies

$$\bar{\theta}(t, y) \equiv \mathbb{E} \left[ \theta(t, y; \tilde{V}) | \mathcal{F}_t^Y \right] = \frac{\frac{\gamma^A \beta \Delta Q}{\lambda} \Pi(t, y) E(t, y) - \Pi(t, y) (1 - \Pi(t, y)) \frac{y - \bar{y}}{nT - t} (E(t, y) - 1)}{\Pi(t, y) \cdot E(t, y) + 1 - \Pi(t, y)}, \quad (43)$$

where  $E(t, y) = e^{-\frac{\gamma^A}{\beta} \lambda y - \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (t-(nT-1))}$ .

The adjusted order flow  $\hat{Y}_{1,t}$  starts from 0 and follows

$$d\hat{Y}_{1,t} = \frac{(\log \tilde{V} - \mu_V) / \left(\frac{\beta-\gamma^A}{\beta} \lambda\right) - \hat{Y}_{1,t}}{nT - t} dt + dZ_t, \quad (44)$$

where  $\tilde{V} = H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  when informed and  $\tilde{V} = \bar{V}$  when not informed.

The above theorem shows that the benchmark's main results still hold when I extend the model with the potential better-informed insider.<sup>37</sup> Conditional on the insider is better

<sup>37</sup>When the insider is always better informed, i.e.,  $\pi_{nT-1} = 1$ , the equilibrium goes back to the benchmark model in section 4. When the market makers are risk-neutral, the equilibrium goes back to Li (2013).

informed, the equilibrium price dynamics is a submartingale when the market makers are risk-compensated. Both the growth rate of the expected pre-FOMC announcement drift and the expected insider's order rate are time-varying, which are caused by the dynamics of the probability estimate. In addition, the pricing rule is nonlinear and stochastic, which drives price volatility, market depth, and price response to be stochastic.

### 5.3 Properties of equilibrium and model calibration

The stochastic pricing rule in Theorem 2 gives me the hope to match the nonlinear pre-FOMC announcement drift in the data. In this section, I study the equilibrium properties and calibrate the model to the pre-FOMC drift that occurs 24 hours before announcements.

**Proposition 3.** *For any smooth distribution of the prior  $G(\pi_{nT-1})$ , the average realized pre-FOMC announcement drift just before announcements ( $t = nT^-$ ) is*

$$\log \mathbb{E} \left[ \frac{P_{nT^-}}{P_{nT-1}} \right] = \eta \gamma^A \beta \Delta Q,$$

where the expectation is taken over all states of nature and  $\eta$  is the fraction of insider that is informed. Meanwhile, the average uncertainty reduction just before announcements is

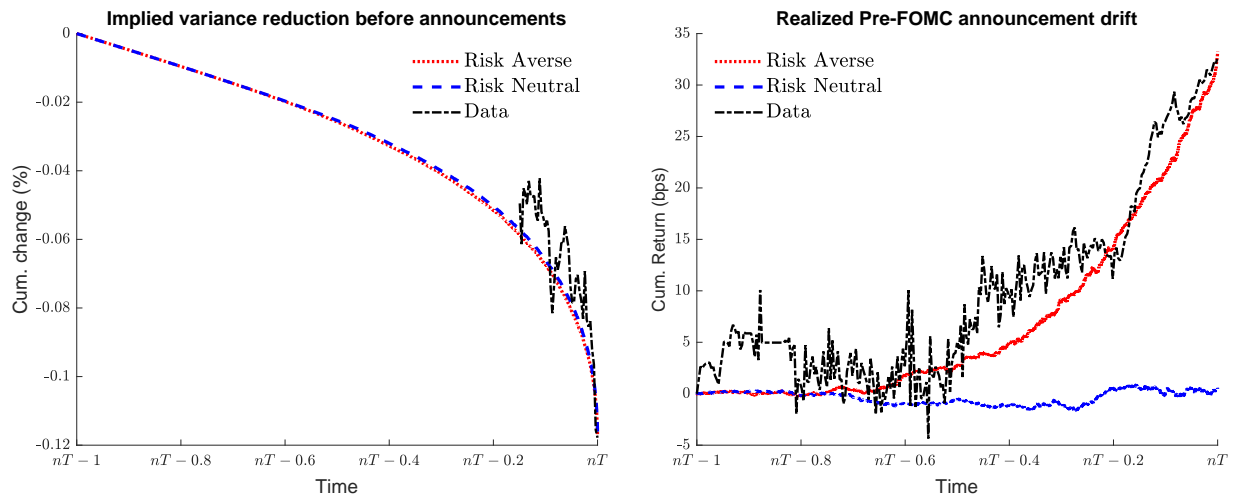
$$\mathbb{E} \left[ \text{Var} \left[ \log P_{nT^-} | \mathcal{F}_{nT^-}^Y \right] - \text{Var} \left[ \log P_{nT^-} | \mathcal{F}_{nT-1}^Y \right] \right] = -\eta \beta^2 \Delta Q.$$

Here the expectation is also taken over all states of nature.

Proposition 3 captures the average realized pre-FOMC drift and the average uncertainty reduction just before announcements in the presence of the potential better-informed insider. The intuition is as follows. When the insider is not informed, the market makers figure that out just before announcement (i.e.,  $\lim_{t \rightarrow nT^-} \Pi(t, \hat{Y}_{1,t}) = 0$ ) and the price  $P_{nT^-}$  equals  $P_{nT-1}$  almost surely. Besides, there is no uncertainty reduction since the insider has no information other than what the market makers have at  $t = nT - 1$ . While when the insider is better informed, all of the private information is eventually incorporated into the price, which is associated with uncertainty reduction. The probability estimate converges to 1 and the price converges to  $A(\hat{m}_{nT}, nT)$  almost surely upon announcements.

The closed-form solutions in Theorem 2 and Proposition 3 generate a precise mapping from the model’s parameters to the asset market evidence before FOMC announcements. The calibration is summarized in Table 1. I choose the fraction of insider that is informed  $\eta$  across these FOMC announcements to be 0.5, consistent with Figure 2. Besides, I assume the market makers prior  $\pi_0 \equiv 0.2$  to match the nonlinear trend of the pre-FOMC announcement drift.<sup>38</sup> I set  $\sigma_m = 0.43\%$  to match the level of the average cumulative pre-FOMC announcement excess return. All other parameters are the same as the benchmark.

**Figure 7: Model implications: uncertainty reduction and the pre-FOMC drift**



This figure plots the average uncertainty reduction and the average realized pre-FOMC drift 24 hours before announcements in the model and in the data. The dotted red lines and the dashed blue lines indicate the cases with the risk-averse market makers and the risk-neutral market makers, respectively. The black lines show the change of  $VIX^2$  and the cumulative return of E-mini around FOMC announcements in the data. The parameters are reported in Table 1.

Figure 7 plots the average uncertainty reduction and the average realized pre-FOMC drift 24 hours before announcements in the model and in the data. The black lines represent the fluctuations in the data. To examine the overnight price dynamics, I calculate the pre-FOMC drift by E-mini instead of the S&P 500 index. VIX is only allowed to trade during regular trading hours between 9:30 a.m. and 4:15 p.m. ET before April 2016.<sup>39</sup> There is no overnight

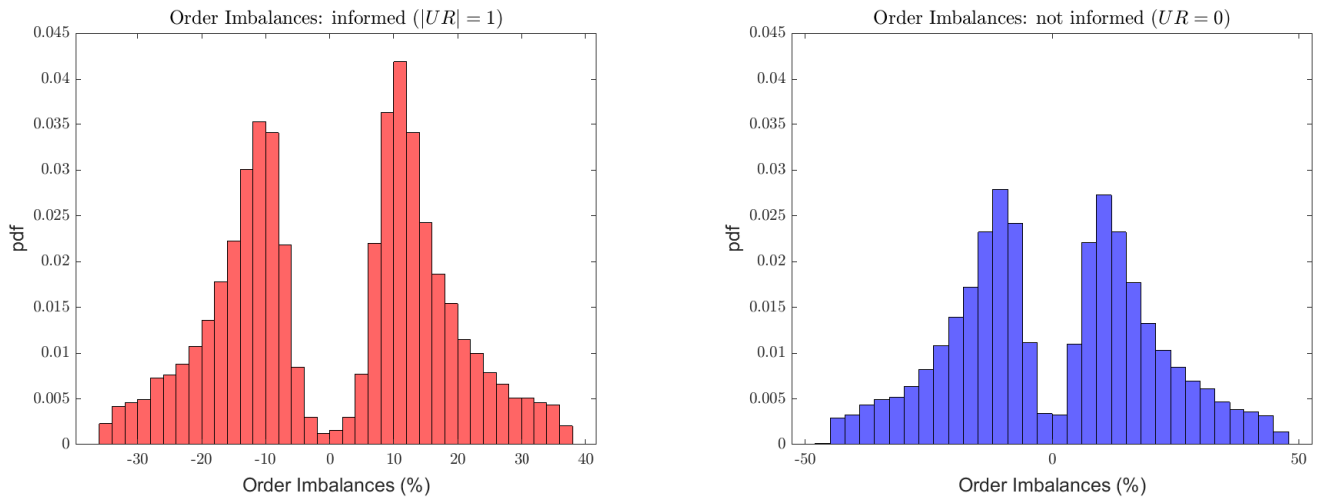
<sup>38</sup>Note that the evolution of the probability estimation is endogenously determined in the model, which affects the trend of the pre-FOMC announcement drift.

<sup>39</sup>In April 2016, Cboe began dissemination of the VIX Index outside of U.S. trading hours so that it can be traded during “extended trading hours” between 3 a.m. and 9:15 a.m ET.

data. Thus, I plot the implied variance reduction from 3 hours before announcements comparing to that 24 hours before FOMC announcements.<sup>40</sup>

The dotted red lines indicate the case under calibrated parameters. For comparison, I also show the case under the risk-neutral market makers (the dashed blue lines) and keep other parameters the same. Both cases match the uncertainty reduction pretty well. However, only under the risk-averse market makers, there is a strictly positive pre-FOMC announcement drift that is consistent with the data. Besides, the dotted red line matches the nonlinear trend of the pre-FOMC drift pretty well that the cumulative return grows faster when approaching FOMC announcements. The intuition behind this is the following. When the insider is informed, as time goes by, the cumulative order flow reveals more private information that speeds up the probability estimation of the market makers. It results in a faster uncertainty reduction, which is associated with the deeper pre-FOMC announcement drift.

**Figure 8:** Model implications: order imbalances



This figure shows the distribution of the order imbalances in the model conditional on whether the insider is informed or not. I simulate 50,000 times in the frequency of one minute and winsorize the order imbalances at the 10th and 90th percentiles to mitigate the effect of outliers. The parameters are reported in Table 1.

I follow equation (33) to calculate the order imbalances in the model. Figure 8 plots the distribution conditional on whether the insider is informed or not. When the insider is

<sup>40</sup>I can extend the implied variance change to 5 hours before announcements if I delete the FOMC announcements that happen at 12:30 p.m. ET. The same pattern holds.



informed, the average of the order imbalances is significantly positive due to the positive excess return before announcements. When the insider is not informed, the average is not significantly different from zero. These model implications are consistent with the empirical facts shown in Figure 3.

## 6 Further implications

In addition to the uncertainty reduction before announcements shown in section 2, I demonstrate that the asset market fluctuations around FOMC announcements are consistent with the model's predictions. First, I provide empirical support that the pre-FOMC announcement drift is consistent with the risk-reduction explanation before announcements from private information in my model, instead of unexpectedly good news. Second, I explain the timing of pre-FOMC drift that occurs 24 hours before announcements when private information is probably known way before. Third, I explain the two distinctive patterns to equity returns on days with an FOMC announcement since April 2011, documented by Boguth, Gregoire, and Martineau (2019). Fourth, I show that my model can reconcile with the auxiliary puzzle documented in Lucca and Moench (2015)—the absence of the pre-FOMC drift in fixed income instruments.

### 6.1 Risk-reduction explanation before FOMC announcements

My model predicts that a more substantial uncertainty reduction is associated with the stronger pre-FOMC drift. In Figure 2, I already show that the pre-FOMC drift only exists when there is uncertainty reduction before announcements. To more formally assess the impact of uncertainty reduction on the excess stock market returns prior to FOMC announcements, I run the following regression

$$\text{Cum. Return}_t = \alpha + \beta \Delta \text{VIX}_t + \varepsilon_t,$$

where both  $\Delta \text{VIX}_t$  and  $\text{Cum. Return}_t$  are calculated from 2 p.m on pre-announcement date to announcement time windows, and  $t$  represents each FOMC announcement. As shown in

Table 5, on average when VIX decreases 1 percent before FOMC news, the cumulative return increases 51.3 basis points.<sup>41</sup> In terms of the high-reduction group, since the constant term  $\alpha$  is not significantly different from zero, the single variable uncertainty reduction can fully account for the pre-FOMC drift, which is consistent with my model.

To understand stock-bond dynamics, Cieslak and Pang (2020) decompose daily innovations in stock returns and yield changes into 4 orthogonal sources of news: growth news (cash-flow risk), monetary news (pure discount-rate risk), hedging premium news (compensation for cash-flow risk), and common premium news (compensation for discount-rate risk). They find risk-premium shocks generate 69% of the average FOMC-day increase (split into 36% and 33% contributions of the common premium and the hedging premium, respectively).<sup>42</sup> This is consistent with my model's mechanism—along with the uncertainty reduction out of insider trading, the pre-FOMC drift is determined by the negative covariance between the innovation to the A-SDF (compensation for discount-rate risk) and the asset value (compensation for cash-flow risk).

The information channel I emphasize is consistent with recent work by Nakamura and Steinsson (2018).<sup>43</sup> They find that Federal Reserve announcements affect beliefs not only about monetary policy but also about economic fundamentals. Both of the two measures of monetary policy surprises constructed by Nakamura and Steinsson (2018) are indifferent from zero on average. Cieslak and Pang (2020) also show that the average growth news component is close to zero and conclude that the FOMC days are not associated with systematically positive or negative news about the economy. Therefore, the pre-FOMC announcement drift can not be driven by unexpectedly good news.

## 6.2 The timing of the pre-FOMC announcement drift

To fully account for the pre-FOMC announcement drift, timing is another puzzle that needs to be explained: Why does it occur 24 hours prior to announcements when private informa-

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<sup>41</sup>This is consistent with the simple dummy variable regression model in Table 6, which indicates the change of VIX before announcements itself can explain a large fraction of the pre-announcement drift.

<sup>42</sup>This number will be higher if they focus on the pre-FOMC drift instead of daily close-to-close returns since there should be no monetary policy shock before announcements.

<sup>43</sup>Jarocinski and Karadi (2020) find similar results—ignoring the central bank information shocks biases the inference on monetary policy nonneutrality. The effect will be stronger if they focus on the pre-FOMC period.

tion is probably known way before?

The insider's expected profit out of the asymmetric information increases in the market's uncertainty when the insider starts to trade. The intuition is that the insider has relatively more private information when the market is noisier. Figure 1 shows the market uncertainty increases from Day -3 and peaks around 24 hours prior to FOMC announcements.<sup>44</sup> However, she can not trade too late since she needs a large amount of liquidity trading to hide her private information. Otherwise, the profit will be smaller. Therefore, to maximize the profit out of the private information, my model implies that the insider starts to trade around 24 hours before announcements, even the insider is probably informed way before. Given the profit estimation of insider trading around FOMC announcements in the literature, the expected profit reduces substantially if the insider trades earlier.<sup>45</sup>

### 6.3 Two distinctive patterns to equity returns: press conferences

Since April 2011, the Chair of the FOMC has been giving a press conference at every other FOMC meeting.<sup>46</sup> At these meetings the FOMC also releases the summary of its members' economic projections (SEP), so that three forms of communication take place: the FOMC statement, the SEP, and the press conference with the Chair.

Boguth, Gregoire, and Martineau (2019) study the impact of the press conferences and find that the pre-FOMC drift is limited to announcements with press conferences since April 2011. Figure 9 shows the average cumulative return on the S&P 500 index on two-day windows with and without press conferences from April 2011.<sup>47</sup> The left top panel shows the VIX index with press conferences decreases significantly before announcements with an average 25.4 basis points pre-FOMC return. Besides, the VIX index keeps decreasing after FOMC announcements indicating the high uncertainty associated with press conferences.

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<sup>44</sup>Consistent with the empirical fact, the uncertainty keeps increasing before announcements as long as the insider has not started to trade in the model.

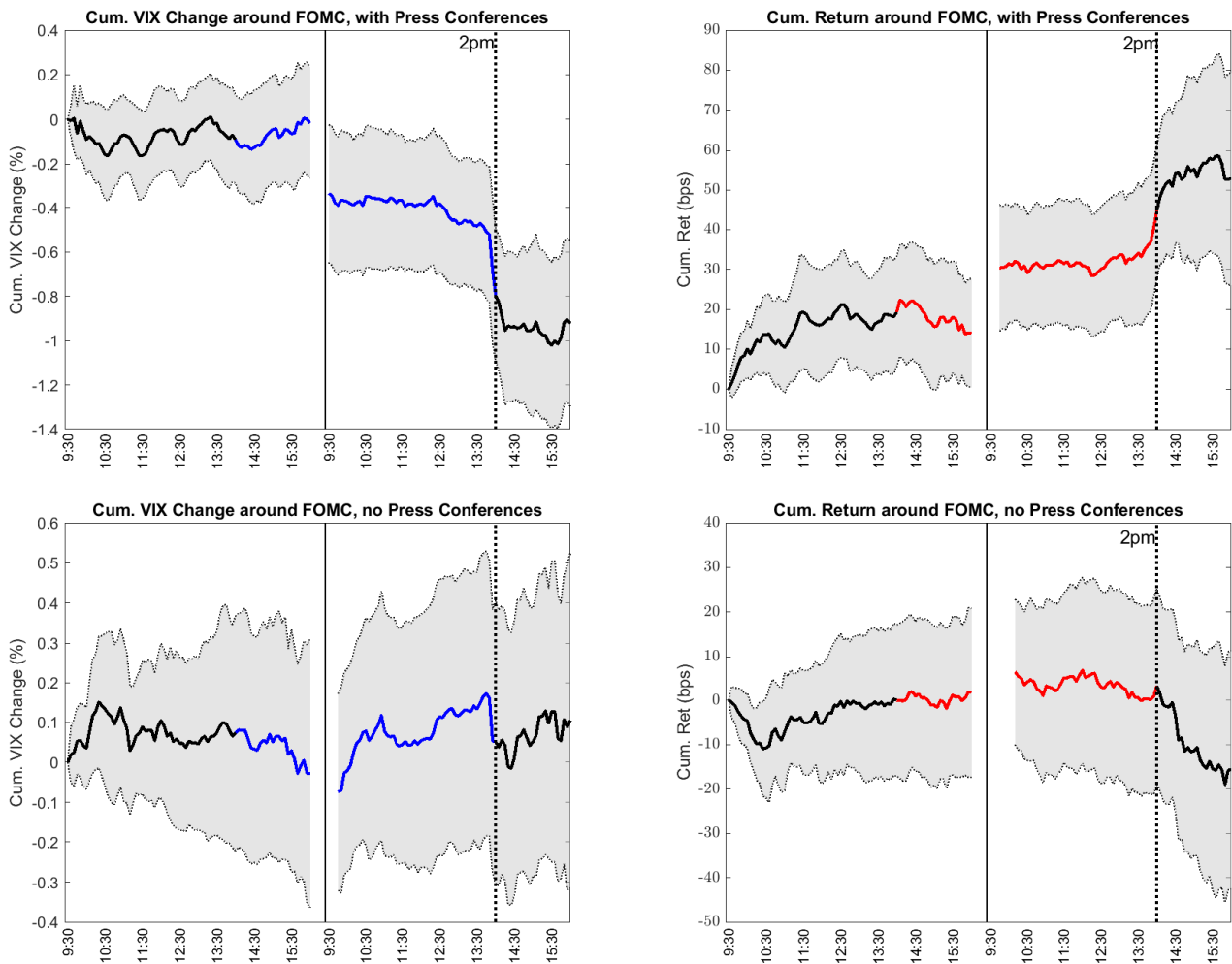
<sup>45</sup>For example, Bernile, Hu, and Tang (2016) estimates the insider's profit ranges between \$2.5 million and \$207.5 million on the S&P 500 futures market alone during the 30-minutes window before FOMC announcements.

<sup>46</sup>From January 2019, the Chairman of the Federal Reserve holds a press conference after each meeting.

<sup>47</sup>From April 2011, some FOMC announcements happen before 2 p.m. The same pattern holds when I precisely capture the timing of all FOMC announcements.

However, the return before announcements without press conferences is not significantly different from zero. Meanwhile, the VIX index almost does not change before and after announcements.

**Figure 9: Classifications of FOMC meetings: press conferences**



This figure shows the average cumulative return on the S&P 500 index on two-day windows with and without press conferences from April 2011 to December 2019. The blue (red) solid line indicates the VIX change (cumulative return of the SPX) on the 2 p.m.-to-2 p.m. pre-FOMC window.

The two distinctive patterns emphasize only when the upcoming FOMC announcements are informative (i.e., with significant uncertainty resolution), there is a pre-FOMC drift.<sup>48</sup> It

<sup>48</sup>These results are consistent with findings in Boguth, Gregoire, and Martineau (2019) that announcements

agrees with my model that the asymmetric information is less influential when the FOMC news is not informative.<sup>49</sup>

## 6.4 The absence of the pre-FOMC drift in fixed income instruments

In this section, I show my model can explain the apparent lack of the pre-FOMC drift in fixed income instruments. In the model, there is a positive (negative) pre-announcement drift if the risk exposure  $\beta$  of the asset to the underlying fundamental is positive (negative). In other words, the sign of the pre-FOMC drift in the model depends on whether the asset is risky or a hedge.

Campbell, Sunderam, and Viceira (2017) and Campbell, Pflueger, and Viceira (2020) document that nominal Treasury bonds changed from risky (positively correlated with stocks) in the 1980s and 1990s to safe (negatively correlated with stocks) in the first decade of the 2000s. The average of time-varying betas of nominal bond is close to zero from 1996 to 2019, which results in the absence of the pre-FOMC drift in fixed income instruments. My results agree with Cieslak and Pang (2020), which find the reduction in the common premium is offset by a decline in the value of the hedging premium, making the overall bond market response economically small and statistically insignificant on FOMC days.

## 7 Discussion

Section 2 provides empirical support and discusses potential sources of private information prior to FOMC announcements. In this section, I discuss the challenges that the private information explanation faces in the literature. After that, I talk about other explanations.

### 7.1 Challenges of the private information explanation

Most mentioned challenges of the private information in the literature are related to consistently positive FOMC news, such as Luuca and Moench (2015), Bilyi (2018), Laarits (2019),

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on days without press conferences convey less price-relevant information.

<sup>49</sup>This explanation agrees with Kurov, Wolfe, and Gilbert (2020). They conclude the weaken of the average pre-FOMC drift since April 2011 comes from the lower uncertainty.

and Cocoma (2020). However, this paper studies the resolution of uncertainty via private information results in an upward drift in market prices even if the private news is on average neutral. Therefore, their arguments do not apply to this framework. For example, they argue that if the drift is caused by private information, the realized pre-announcement return should predict with a positive sign that the market response to the announcement. While my model predicts there is no correlation between the pre-FOMC returns and announcement returns, which is supported in Lucca and Moench (2015).<sup>50</sup>

Besides, as shown in section 5, the model only requires that the insider is informed for *some* FOMC announcements instead of all of them. This is consistent with Figure 2 that uncertainty reduction only happens before some FOMC announcements, which is associated with the pre-FOMC drift.

## 7.2 Other explanations

Hu, Pan, Wang, and Zhu (2020) and Laarits (2019) contribute the pre-FOMC drift to uncertainty reduction before FOMC news in a representative-agent framework. Though their stories are risk-based as this paper, there are two main differences: (1) The market news carries two different types of risks, and only one type of risk is resolved before FOMC announcements. (2) All investors observe the resolved information at the same time, i.e., there is no asymmetric information caused by private information.<sup>51</sup> Both papers face the challenge of explaining that there is no uncertainty resolution for some FOMC announcements and the pre-FOMC drift's timing. In addition to that, their models predict substantial post-announcement returns when the other risk is resolved. However, the post-FOMC announcement return is not significantly different zero, as documented in Lucca and Moench (2015).

To account for the pre-FOMC drift, Cocoma (2020) studies a model where both the risk

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<sup>50</sup>Section 24.5 of Back (2017) shows the insider trades more slowly than in the standard model since the insider considers the effect of her trades on the price. This can potentially explain why the trading volume is lower before FOMC announcements.

<sup>51</sup>The two risks are different in these two papers. In Hu, Pan, Wang, and Zhu (2020), the uncertainty about the potential magnitude of the news' market impact is resolved before announcements, while the risk associated with the news realization itself is resolved upon announcements. Laarits (2019) assume there are two types of announcements the Fed will make, which will reveal either monetary policy stance or long-term growth expectations. All investors learn the type of announcements before FOMC news, which resolves part of the uncertainty.

and disagreement are very low before announcements and very high after announcements. However, the risk pattern is the opposite of Hu, Pan, Wang, and Zhu (2020), that find the risk (measured by the VIX index) starts to increase six days before announcements, then decreases from 24 hours before FOMC news until the end of days with announcements. Also, Ying (2020) shows that both call and put open interest decrease significantly at the end of days with announcements, which can not be explained by higher disagreement after FOMC news.

## 8 Conclusion

The substantial stock market return before announcements when the FOMC members refrain from discussions of monetary policy information provides a notable challenge to standard asset pricing theory. In this paper, I propose and test the private information explanation to account for the pre-FOMC announcement drift. When the uncertainty reduces before FOMC, the abnormal order imbalances are 1.85-2.17% higher in the direction of the realized return in the 24-hour window before FOMC announcements. It provides evidence consistent with informed trading when the pre-FOMC drift occurs. I integrate Kyle's (1985) model into a standard consumption-based asset pricing framework where the market makers require compensation for the risk of assets' fundamentals. Insider trading resolves uncertainty gradually and results in an upward drift in market prices, even the private news is on average neutral. The limit of the equilibrium is the traditional Kyle model as the market makers converge to be risk-neutral. The convergence result demonstrates a strictly positive pre-FOMC drift if and only if the market makers are compensated for the risk of assets' fundamentals.

This paper provides a general framework to account for other pre-event drifts. A large group of papers treats average abnormal positive excess returns before events as evidence of insider trading and tests the market liquidity measure inspired by Kyle. While in the standard Kyle-type models, the expected average excess return before announcements is zero due to the risk-neutral market makers. Therefore, this paper provides a general theoretical framework for other pre-event drifts if the risk of the news is priced in the pricing

kernel. The different equilibrium implications comparing to the standard Kyle-type models offer new insights into how private news affects asset prices, volatility, volume, and market liquidity. I leave these interesting directions for future work.



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**Table 1: Parameters**

The model is calibrated at annually frequency. I assume the prescheduled announcements happen at the monthly frequency, that is,  $T = \frac{1}{12}$ .

Parameter	symbol	value
<i>Aggregate output</i>		
long run output growth rate	$\bar{m}$	1.50%
volatility of aggregate consumption	$\sigma_C$	3.16%
persistence of the AR(1) process	$a_m$	4.5%
volatility of the AR(1) process (benchmark)	$\sigma_m$	0.30%
volatility of the AR(1) process (full model)	$\sigma_m$	0.43%
<i>Uncertainty and asset value</i>		
the transparency of announcements	$\sigma_s^2$	$1.6 \times 10^{-5}$
the exposure of the risky asset	$\beta$	3
<i>Preference</i>		
risk aversion	$\gamma$	6.6
elasticity of intertemporal substitution	$\psi$	1
subjective discount factor	$\rho$	0.005
<i>Parameters in the full model</i>		
prior of the probability that the insider is informed	$\pi_{nT-1}$	0.2
fraction of the informed insider across announcements	$\eta$	0.5

**Table 2: Summary Statistics on S&P500 Index Excess Returns and Changes in VIX.**

Note: This table reports summary statistics for the pre-announcement day 2 p.m (-1) – announcement (ann) and announcement – close changes in VIX ( $\Delta VIX$ ) and cumulative excess returns on the S&P500 (*Cum.Return*). The close time is 3:55 p.m. The samples are: (1) All FOMC announcements, (2 and 3) FOMC announcements sorted on uncertainty, which is first and third tertiles of changes in VIX ( $\Delta VIX_{t-1}$ ) between open and 2 p.m on pre-announcement dates, and (4 and 5) FOMC announcements with and without FOMC Press Conference. “Sharpe ratio” is the annualized Sharpe ratio on FOMC announcement returns. The sample period is from 1996:01 to 2019:11, and from 2011:04 for press conference sample. “No. of FOMC” is the number of FOMC in each subset. t-statistics for the mean are reported in parentheses. \*\*\*, \*\*, and \* represent 1%, 5%, and 10%.

		(1) All	Sort on Uncertainty		Press Conference	
			(2) High	(3) Low	(4) Yes	(5) No
$\Delta VIX$ (%)	2 p.m (-1)-ann	-0.300*** (-3.402)	-1.459*** (-14.020)	0.748*** (5.485)	-0.734*** (-5.213)	-0.072 (-0.465)
	ann-close	-0.318*** (-4.405)	-0.246* (-1.819)	-0.487*** (-3.288)	-0.203* (-1.773)	-0.035 (-0.181)
	No. of FOMC	187	61	63	34	35
	2 p.m (-1)-ann	0.332*** (5.636)	0.944*** (9.016)	-0.154 (-1.671)	0.254*** (3.314)	0.014 (0.158)
Cum.Return (%)	ann-close	-0.030 (-0.446)	-0.064 (-0.478)	0.063 (0.603)	0.099 (1.090)	-0.157 (-1.328)
	No. of FOMC	187	61	63	34	35

**Table 3: Compare the order imbalances conditional on FOMC announcement indicators.**

Note: This table compares the level of order imbalances of the E-mini Standard & Poor's 500 futures in the 24-hour window before FOMC announcements. *OIN* is the order imbalance defined as  $\frac{B-S}{B+S}$ , where *B* (*S*) is the aggregate buyer-initiated (seller-initiated) trading volume as measured by number of trades. *OID* is calculated similarly using dollar trading volume. For every FOMC announcement, I calculate the average level of order imbalances in the 24-hour window before FOMC with and without uncertainty reduction. Column (1) reports the average level of order imbalances on announcements with the pre-FOMC uncertainty reduction ( $UR = \pm 1$ ). Column (2) reports the average level of order imbalances on announcements without the pre-FOMC uncertainty reduction ( $UR = 0$ ). Column (3) reports the difference between columns (1) and (2). The sample period is from 1996:01 to 2019:11. t-statistics are reported in parentheses. \*\*\*, \*\*, and \* represent 1%, 5%, and 10%.

	$UR = \pm 1$	$UR = 0$	Difference
	(1)	(2)	(3)
<i>OIN</i> (%)	1.272***	-0.719***	1.991***
	(4.22)	(-2.91)	(5.11)
<i>OID</i> (%)	1.789***	-1.056***	2.845***
	(4.21)	(-3.13)	(5.24)

**Table 4: Order imbalances conditional on FOMC announcement indicators.**

Note: This table reports ordinary least squares estimates of the relation between event-time order imbalances in the E-mini Standard & Poor's 500 futures market and announcement day indicators. For each FOMC announcement, the sample includes the announcement day ( $ANN = 1$ ) and non-announcement days in the prior 21 trading days or since the last announcement ( $ANN = 0$ ).  $OIN$  is the order imbalance defined as  $\frac{B-S}{B+S}$ , where  $B$  ( $S$ ) is the aggregate buyer-initiated (seller-initiated) trading volume as measured by number of trades.  $OID$  is calculated similarly using dollar trading volume. Both dependent variables are calculated in three event windows:  $[-24H, 0]$ ,  $[-24H, -12H]$ , and  $[-12H, 0]$ , where zero is the official release time of the FOMC announcement and the time unit is an hour. The uncertainty-reduced indicator,  $UR$ , is equal to one (negative one) for announcements that the pre-FOMC realized return is positive (negative) under uncertainty reduction before FOMC and zero otherwise. The sample period is from 1996:01 to 2019:11. t-statistics are reported in parentheses. \*\*\*, \*\*, and \* represent 1%, 5%, and 10%.

	[-24H, 0]		[-24H, -12H]		[-12H, 0]	
	(1) $OIN$	(2) $OID$	(3) $OIN$	(4) $OID$	(5) $OIN$	(6) $OID$
Constant	-0.047 (-0.98)	-0.154** (-2.29)	0.210** (2.23)	-0.065 (-0.49)	-0.104 (-1.52)	-0.182** (-1.98)
$ANN$	-0.222 (-0.87)	-0.073 (-0.20)	-0.562 (-1.11)	-0.319 (-0.45)	-0.172 (-0.47)	-0.019 (-0.04)
$UR$	1.851*** (5.61)	2.167*** (4.60)	2.097*** (3.20)	2.213** (2.39)	1.694*** (3.55)	2.052*** (3.21)



**Table 5: Returns on the S&P500 Index.**

Note: This table shows results for regressing the changes in VIX ( $\Delta VIX$ ) on the cumulative excess returns on the S&P500 (*Cum.Return*),  $Cum.Return_t = \alpha + \beta\Delta VIX_t + \varepsilon_t$  where both  $\Delta VIX_t$  and  $Cum.Return_t$  are calculated from 2 p.m on pre-announcement date to 2 p.m on announcement date windows, and  $t$  represents each FOMC announcement. The samples are: (1) All FOMC announcements, (2 and 3) FOMC announcements sorted on uncertainty, which is first and third tertiles of changes in VIX ( $\Delta VIX_{t-1}$ ) between open and 2 p.m on pre-announcement dates, and (4 and 5) FOMC announcements with and without FOMC Press Conference. The sample period is from 1996:01 to 2019:11, and from 2011:04 for press conference sample. "Obs." and "No. of FOMC" are the number of observations and amount of FOMC in each subset, respectively. t-statistics are reported in parentheses. \*\*\*, \*\*, and \* represent 1%, 5%, and 10%.

	(1) All	Sort on Uncertainty		Press Conference	
		(2) High	(3) Low	(4) Yes	(5) No
$\Delta VIX$	-0.513*** (-16.387)	-0.603*** (-5.775)	-0.493*** (-8.405)	-0.346*** (-4.646)	-0.460*** (-7.672)
Constant	0.170*** (4.374)	0.056 (0.319)	0.207*** (2.700)	-0.002 (-0.031)	-0.021 (-0.390)
Obs.	187	61	63	34	35
No. of FOMC	187	61	63	34	35

**Table 6: S&P500 Index Return Time-Series Regressions.**

Note: This table reports results for regressions of the time-series of pre-FOMC announcement returns on various explanatory variables for the sample period 1996:01 to 2019:11. The dependent variable is a time-series of cumulative excess returns on the S&P500 from 2 p.m on days before announcement to 2 p.m on days of scheduled FOMC announcements. The first independent variable in Column (1) and (2) is Pre-FOMC dummy ( $D_{\text{FOMC}}$ ), which is equal to one when a scheduled FOMC announcement has been released in the following 24-hour interval and zero otherwise. The second independent variable in Column (2) is the interaction of changes in VIX and Pre-FOMC dummy ( $\Delta\text{VIX} \times D_{\text{FOMC}}$ ). "Sharpe ratio" is the annualized Sharpe ratio on FOMC announcement returns. "Obs." and "No. of FOMC" are the number of observations and amount of FOMC in each subset, respectively. t-statistics are reported in parentheses. \*\*\*, \*\*, and \* represent 1%, 5%, and 10%.

	(1)	(2)
$D_{\text{FOMC}}$	0.314*** (3.737)	0.160** (1.857)
$\Delta\text{VIX} \times D_{\text{FOMC}}$		-0.513*** (-7.507)
Constant	0.010 (0.671)	0.010 (0.674)
Sharpe Ratio	1.14	1.14
Obs.	5899	5899
No. of FOMC	187	187

## APPENDICES

The following appendices provide details of the proof in section 3, 4, and 5. Appendix A shows the related proof of the macroeconomic environment in section 3. Appendix B contains all the proofs for the benchmark economy that the insider is always informed. Appendix C provides the details for the economy that the market makers are uncertain about whether the insider is informed or not.

### A Proof of the standard asset pricing framework

**Proof of Lemma 1.** The HJB equation for recursive utility satisfies

$$\bar{f}(C_t, V(\hat{m}_t, t, C_t)) + \mathcal{L}[(\hat{m}_t, t, C_t)] = 0.$$

Due to homogeneity, consider the value function of the form

$$V(\hat{m}_t, t, C_t) = \frac{1}{1-\gamma} H(\hat{m}_t, t) C_t^{1-\gamma},$$

where  $H(\hat{m}_t, t)$  satisfies the following HJB equation:

$$\begin{aligned} 0 = & \frac{\rho}{1-\frac{1}{\psi}} \left( H(\hat{m}_t, t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} - 1 \right) + \left( \hat{m}_t - \frac{1}{2} \gamma \sigma_C^2 \right) + \frac{1}{1-\gamma} \frac{H_t(\hat{m}_t, t)}{H(\hat{m}_t, t)} \\ & + \left[ \frac{1}{1-\gamma} a_m (\bar{m} - \hat{m}_t) + q_t \right] \frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)} + \frac{1}{2} \frac{1}{1-\gamma} \frac{H_{mm}(\hat{m}_t, t)}{H(\hat{m}_t, t)} \left( \frac{q_t}{\sigma_C} \right)^2. \end{aligned} \quad (\text{A1})$$

with the boundary condition that

$$H(\hat{m}_t^-, t^-) = \mathbb{E} [H(\hat{m}_t^+, t^+) | \hat{m}_t^-, q_t^-]. \quad (\text{A2})$$

The state price process of recursive utility satisfies

$$\frac{d\pi_t}{\pi_t} = \frac{d\bar{f}_C(C_t, V_t)}{\bar{f}_C(C_t, V_t)} + \bar{f}_V(C_t, V_t) dt. \quad (\text{A3})$$

Therefore, for  $n = 1, 2, \dots$ , in the interior of  $(nT, (n+1)T)$ , the law of motion of the state price density,  $\pi_t$  satisfies the stochastic differential equation of the form:

$$d\pi_t = \pi_t [-r(\hat{m}, t) dt - \sigma_\pi(\hat{m}, t) d\tilde{B}_{C,t}],$$

where

$$r(\hat{m}, t) = \rho + \frac{1}{\psi} \hat{m} - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma_C^2 - \frac{\gamma - \frac{1}{\psi}}{1-\gamma} \frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)} q(t) + \frac{1}{2} \frac{\frac{1}{\psi} - \gamma}{1-\gamma} \frac{1 - \frac{1}{\psi}}{1-\gamma} \left( \frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)} \right)^2 \left( \frac{q(t)}{\sigma_C} \right)^2,$$

is the risk-free interest rate, and

$$\sigma_\pi(\hat{m}, t) = \gamma\sigma_C - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)} \frac{q(t)}{\sigma_C},$$

is the market price of the Brownian motion risk.

Upon announcements, the stochastic discount factor for a small interval  $\Delta$  is

$$\text{SDF}_{t,t+\Delta} = e^{-\rho\Delta} \left( \frac{C_{t+\Delta}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{W_{t+\Delta}}{E_t \left[ W_{t+\Delta}^{\frac{1}{1-\gamma}} \right]} \right)^{\frac{1}{\psi} - \gamma}, \quad (\text{A4})$$

where

$$W_t = [(1 - \gamma) V(\hat{m}, t, C)]^{\frac{1}{1-\gamma}}, \quad (\text{A5})$$

which implies

$$\text{SDF}_{t,t+\Delta} = e^{-\rho\Delta} \left( \frac{C_{t+\Delta}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{\left( H(\hat{m}_{t+\Delta}, t + \Delta) C_{t+\Delta}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}}{E_t \left[ H(\hat{m}_{t+\Delta}, t + \Delta) C_{t+\Delta}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}.$$

Therefore, when upon announcements ( $t = nT$ ) and  $\Delta \rightarrow 0$ ,

$$\Lambda_{t,t+\Delta}^* = \frac{[H(\hat{m}_{t+\Delta}, t + \Delta)]^{\frac{1}{\psi} - \gamma}}{[E_t(H(\hat{m}_{t+\Delta}, t + \Delta))]^{\frac{1}{\psi} - \gamma}}. \quad (\text{A6})$$

The term  $\beta H(\hat{m}_t, t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}$  is the consumption-wealth ratio. Consider the following log-linear expansion:  $e^{\ln x} \approx e^{\ln \bar{x}} + e^{\ln \bar{x}} (\ln x - \ln \bar{x})$ ,

$$\beta H(\hat{m}_t, t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} \approx \kappa + \kappa \left[ \ln \rho - \frac{1 - \frac{1}{\psi}}{1 - \gamma} \ln H(\hat{m}_t, t) - \ln \kappa \right],$$

where  $\kappa = \beta H(\bar{m}, t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}$  is the consumption-wealth ratio when  $\hat{m}_t$  is equal to its unconditional mean  $\bar{m}$ . Therefore, I can approximate  $\beta H(\hat{m}_t, t)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}$  as

$$\begin{aligned} \frac{\rho}{1 - \frac{1}{\psi}} \left[ H(\hat{m}_t, t)^{1 - \frac{1-\frac{1}{\psi}}{1-\gamma}} - 1 \right] &\approx \frac{1}{1 - \frac{1}{\psi}} \left[ \kappa + \kappa \left[ \ln \rho - \frac{1 - \frac{1}{\psi}}{1 - \gamma} \ln H(\hat{m}_t, t) - \ln \kappa \right] - \rho \right] \\ &= -\frac{\kappa}{1 - \gamma} \ln H(\hat{m}_t, t) + \xi_0, \end{aligned}$$

where I denote  $\xi_0 \triangleq \frac{1}{1 - \frac{1}{\psi}} [\kappa - \rho - \kappa (\ln \kappa - \ln \rho)]$ .

The HJB equation (A1) is written as

$$\begin{aligned} & \zeta_0 - \frac{\kappa}{1-\gamma} \ln H(\hat{m}_t, t) + \left( \hat{m}_t - \frac{1}{2} \gamma \sigma_C^2 \right) + \frac{1}{1-\gamma} \frac{H_t(\hat{m}_t, t)}{H(\hat{m}_t, t)} \\ & + \left[ \frac{1}{1-\gamma} a_m (\bar{m} - \hat{m}_t) + q_t \right] \frac{H_m(\hat{m}_t, t)}{H(\hat{m}_t, t)} + \frac{1}{2} \frac{1}{1-\gamma} \frac{H_{mm}(\hat{m}_t, t)}{H(\hat{m}_t, t)} \left( \frac{q_t}{\sigma_C} \right)^2 = 0. \end{aligned} \quad (\text{A7})$$

I guess  $H(\hat{m}_t, t)$  is of the form

$$H(\hat{m}_t, t) = e^{-\gamma^A \hat{m}_t + \mathcal{H}(t)}. \quad (\text{A8})$$

Using the method of undetermined coefficients, I have that

$$\gamma^A = \frac{\gamma - 1}{a_m + \kappa}, \quad (\text{A9})$$

$$\mathcal{H}'(t) = \kappa \mathcal{H}(t) - f(t). \quad (\text{A10})$$

where  $f(t)$  is defined as:

$$f(t) = \frac{(1-\gamma)^2}{a_m + \kappa} q(t) + \frac{1}{2} \frac{(1-\gamma)^2}{(a_m + \kappa)^2} \frac{1}{\sigma_C^2} q^2(t) - \frac{1}{2} \gamma (1-\gamma) \sigma_C^2 + a_m \bar{m} \frac{1-\gamma}{a_m + \kappa} + \zeta_0.$$

$\mathcal{H}(t)$  can be solved in closed form from equations (A10) and (A2). In order to solve for asset prices, I do not need the functional form  $\mathcal{H}(t)$ .

Note that this above approximation is *exact* if  $\psi = 1$ , in which case

$$\gamma^A = \frac{\gamma - 1}{a_m + \rho}. \quad (\text{A11})$$

Besides, from equations (A6) and (A9), it is straightforward to show the A-SDF is counter-cyclical if and only if the agent has early resolution of uncertainty, i.e.,  $\gamma > \frac{1}{\psi}$ , which is equivalent to  $\gamma^A > 0$  when  $\psi = 1$ . ■

**Lemma 5.** *Under the assumption that the aggregate consumption does not change in the 24-hour window before announcements, at  $t = nT - 1$ , the agent has a prior that the expected growth rate upon announcements  $\hat{m}_{nT}$  is normally distributed  $N(\hat{m}_{nT-1}, \Delta Q)$  where  $\Delta Q = q_{nT-1} - q_{nT}$ .*

**Proof.** At announcements  $t = nT$ , the agent updates her belief using Bayes' rule:

$$\hat{m}_{nT} = q_{nT} \left[ \frac{1}{\sigma_s^2} s_n + \frac{1}{q_{nT-1}} \hat{m}_{nT-1} \right], \quad \frac{1}{q_{nT}} = \frac{1}{\sigma_s^2} + \frac{1}{q_{nT-1}}, \quad (\text{A12})$$

which implies

$$\mathbb{E}_{nT-1}[\hat{m}_{nT}] = \hat{m}_{nT-1}, \quad \text{Var}_{nT-1}[\hat{m}_{nT}] = \left( \frac{q_{nT}}{\sigma_s^2} \right)^2 (q_{nT-1} + \sigma_s^2) = q_{nT-1} - q_{nT}. \quad (\text{A13})$$

Therefore, at  $t = nT - 1$ ,  $m_{nT}$  is normally distributed  $N(\hat{m}_{nT-1}, \Delta Q)$  where  $\Delta Q = q_{nT-1} - q_{nT}$ . ■

## B Proof of Theorem 1

The proof is in several steps.

### B.1 Step 1: Market Maker's Updating

First, I establish that if the market makers conjecture that the insider's trading strategy follows equation (15), then the price dynamics equation (20) satisfies the market makers' break-even pricing rule given in equation (10).

**Proof of Lemma 2.** The conjectured trading strategy (15) implies that

$$\begin{aligned}
\theta_t &= \frac{\log [A(\hat{m}_{nT}, nT)] - \mu_P + \gamma^A \beta \Delta Q}{(nT - t) \lambda} - \frac{Y_t}{nT - t} \\
&= \frac{\frac{\beta - \gamma^A}{\beta} \{ \log [A(\hat{m}_{nT}, nT)] - \mu_P + \gamma^A \beta \Delta Q \}}{(nT - t) \left( \frac{\beta - \gamma^A}{\beta} \lambda \right)} - \frac{Y_t}{nT - t} \\
&= \frac{\left( \log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \left( \mu_V - \frac{\gamma^A (\beta - \gamma^A)}{\beta^2} \sigma_v^2 \right) \right) / \left( \frac{\beta - \gamma^A}{\beta} \lambda \right) - Y_t}{nT - t}, \tag{A14}
\end{aligned}$$

where the last equality comes from  $H(\hat{m}_{nT}, nT) = e^{-\gamma^A \hat{m}_{nT} + \mathcal{H}(nT)}$  and  $A(\hat{m}_{nT}, nT) = e^{\beta \hat{m}_{nT} + N(nT)}$ . Here  $\mu_P = \beta \hat{m}_{nT-1} + N(nT)$  and  $\mu_V = (\beta - \gamma^A) \hat{m}_{nT-1} + \mathcal{H}(nT) + N(nT)$ .

Therefore, the aggregate trading volume follows

$$dY_t = \theta_t dt + dZ_t = \frac{\left( \log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \left( \mu_V - \frac{\gamma^A (\beta - \gamma^A)}{\beta^2} \sigma_v^2 \right) \right) / \left( \frac{\beta - \gamma^A}{\beta} \lambda \right) - Y_t}{nT - t} dt + \sigma_z dB_t, \tag{A15}$$

where  $dZ_t = \sigma_z dB_t$  and  $Y_{nT-1} = 0$ .

Now let me define the observation and innovation process. Set  $Y_{nT-1}^* = 0$  and

$$\begin{aligned}
dY_t^* &= \frac{1}{\sigma_z} \left( dY_t + \frac{\left( \mu_V - \frac{\gamma^A (\beta - \gamma^A)}{\beta^2} \sigma_v^2 \right) / \left( \frac{\beta - \gamma^A}{\beta} \lambda \right) + Y_t}{nT - t} dt \right) \\
&= \frac{\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]}{\frac{\beta - \gamma^A}{\beta} \sigma_v (nT - t)} dt + dB_t,
\end{aligned}$$

where the last equality comes from  $\lambda = \frac{\sigma_v}{\sigma_z}$ . Because  $Y_t$  are observable to market makers,  $Y^*$  is also observable. The corresponding innovation process is given by

$$dB_t^* = \frac{\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \hat{v}_t}{\frac{\beta - \gamma^A}{\beta} \sigma_v (nT - t)} dt + dB_t$$

where

$$\hat{v}_t = \mathbb{E} \left[ \log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] \mid \mathcal{F}_t^Y \right]. \quad (\text{A16})$$

The Kalman-filter equation implies

$$d\hat{v}_t = \frac{\Sigma_{v,t}}{\frac{\beta - \gamma^A}{\beta} \sigma_v (nT - t)} dB_t^*, \quad (\text{A17})$$

where

$$\Sigma_{v,t} = \text{Var} \left[ \log H(\hat{m}_{nT}, nT) v(\hat{m}_{nT}, nT) \mid \mathcal{F}_t^Y \right], \quad (\text{A18})$$

is the conditional variance of  $\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]$  given market maker's information (on the filtration  $\mathcal{F}_t^Y$ ). The Kalman-filter equation also implies the dynamics of the posterior variance:

$$\frac{1}{\Sigma_{v,t}} = \frac{1}{\Sigma_{v,0}} + \int_{nT-1}^t \frac{1}{(nT-s)^2 \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2} ds = \frac{1}{\left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2} + \frac{t - (nT - 1)}{(nT - t) \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2} = \frac{1}{(nT - t) \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2}, \quad (\text{A19})$$

which implies

$$\Sigma_{v,t} = \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2 (nT - t). \quad (\text{A20})$$

Thus, the filtering equation (A17) is

$$d\hat{v}_t = \frac{\beta - \gamma^A}{\beta} \sigma_v dB_t^* = \frac{\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \hat{v}_t}{nT - t} dt + \frac{\beta - \gamma^A}{\beta} \sigma_v dB_t. \quad (\text{A21})$$

Define an adjusted order flow  $\hat{Y}_t$  as

$$\hat{Y}_t \equiv Y_t - \int_{nT-1}^t \left( \frac{\gamma^A \beta \Delta Q}{\lambda} \right) ds = Y_t - \frac{\gamma^A \beta \Delta Q}{\lambda} [t - (nT - 1)]. \quad (\text{A22})$$

From the aggregate trading volume (A15), the adjusted order flow follows

$$\begin{aligned} d\hat{Y}_t &= dY_t - \frac{\gamma^A \beta \Delta Q}{\lambda} dt \\ &= \frac{(\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \mu_V) / \left(\frac{\beta - \gamma^A}{\beta} \lambda\right) - \hat{Y}_t}{nT - t} dt + \sigma_z dB_t. \end{aligned} \quad (\text{A23})$$

This implies

$$\frac{\beta - \gamma^A}{\beta} \lambda d\hat{Y}_t = \frac{\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \left[ \mu_V + \frac{\beta - \gamma^A}{\beta} \lambda \hat{Y}_t \right]}{nT - t} dt + \frac{\beta - \gamma^A}{\beta} \sigma_v dB_t. \quad (\text{A24})$$

Since  $\hat{v}_{nT-1} = \mathbb{E} [\log H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1}^Y] = \mu_V$  and  $\hat{Y}_{nT-1} = 0$ , combining (A21) and (A24) gives,

$$d\hat{v}_t = \frac{\beta - \gamma^A}{\beta} \sigma_v dB_t^* = \frac{\beta - \gamma^A}{\beta} \lambda d\hat{Y}_t = \frac{\beta - \gamma^A}{\beta} \lambda \left[ dY_t - \frac{\gamma^A \beta \Delta Q}{\lambda} dt \right], \quad (\text{A25})$$

where the last equality holds due to the definition of the adjusted order flow in equation (A22). From the filtering theory,  $B_t^*$  is a standard Brownian Motion with respect to market makers' filtration. Therefore, the adjusted order flow  $\hat{Y}_t$  is a Brownian Motion with instant variance  $\sigma_z^2$  under  $\mathcal{F}_t^Y$ . This also implies

$$\mathbb{E} [\theta_t | \mathcal{F}_t^Y] = \frac{\gamma^A \beta \Delta Q}{\lambda}, \quad (\text{A26})$$

is the market makers' expectation of the insider's order rate, which is strictly positive when the market makers are risk compensated.

The market makers' prior about  $\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]$  at time  $nT - 1$  is represented by a normal distribution. The Kalman filter implies the posterior distribution of  $\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]$  under  $\mathcal{F}_t^Y$  is also Gaussian, which is summarized by the posterior mean  $\hat{v}_t$  and the posterior variance  $\Sigma_{v,t}$ . Therefore,  $\forall t \in [nT - 1, nT]$ , the market makers' estimation of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  is

$$\begin{aligned} V_t &= \mathbb{E} \left[ H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right] \\ &= \mathbb{E} \left[ e^{\log H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)} | \mathcal{F}_t^Y \right] \\ &= e^{\hat{v}_t + \frac{1}{2} \Sigma_{v,t}} = e^{\hat{v}_t + \frac{1}{2} (nT-t) \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2}. \end{aligned} \quad (\text{A27})$$

Applying Ito's Lemma, from equation (A25), I find

$$\begin{aligned} \frac{dV_t}{V_t} &= \frac{1}{V_t} \left[ V_t d\hat{v}_t + \frac{1}{2} V_t (d\hat{v}_t)^2 - \frac{1}{2} \sigma_v^2 V_t dt \right] \\ &= d\hat{v}_t = \frac{\beta - \gamma^A}{\beta} \lambda d\hat{Y}_t. \end{aligned} \quad (\text{A28})$$



Similarly, I define  $\Lambda_t^*$  as the posterior mean of  $\log H(\hat{m}_{nT}, nT)$  under market makers' information:

$$\begin{aligned}
\Lambda_t^* &= \mathbb{E} \left[ \log H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right] \\
&= -\gamma^A \mathbb{E} \left[ \hat{m}_{nT} | \mathcal{F}_t^Y \right] + \mathcal{H}(nT) \\
&= \frac{-\gamma^A}{\beta - \gamma^A} \mathbb{E} \left[ (\beta - \gamma^A) \hat{m}_{nT} | \mathcal{F}_t^Y \right] + \mathcal{H}(nT) \\
&= \frac{-\gamma^A}{\beta - \gamma^A} \mathbb{E} \left[ \log H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right] + \frac{\beta \mathcal{H}(nT) + \gamma^A N(nT)}{\beta - \gamma^A} \\
&= \frac{-\gamma^A}{\beta - \gamma^A} \hat{v}_t + \frac{\beta \mathcal{H}(nT) + \gamma^A N(nT)}{\beta - \gamma^A}
\end{aligned} \tag{A29}$$

where the last equality holds due to equation (A16). It implies

$$d\Lambda_t^* = \frac{-\gamma^A}{\beta - \gamma^A} d\hat{v}_t,$$

with  $\Lambda_{nT-1}^* = -\gamma^A \hat{m}_{nT-1} + \mathcal{H}(nT)$ . Therefore, the posterior variance of  $\log H(\hat{m}_{nT}, nT)$  under the market makers' information is,

$$\Sigma_{\Lambda^*,t} = \text{Var} \left[ \log [H(\hat{m}_{nT}, nT)] | \mathcal{F}_t^Y \right] = \left( \frac{\gamma^A}{\beta - \gamma^A} \right)^2 \Sigma_{v,t} = \left( \frac{\gamma^A}{\beta} \right)^2 (nT - t) \sigma_v^2.$$

The Kalman filter implies the posterior distribution of  $\log [H(\hat{m}_{nT}, nT)]$  under  $\mathcal{F}_t^Y$  is also Gaussian, which is summarized by the posterior mean  $\Lambda_t^*$  and the posterior variance  $\Sigma_{\Lambda^*,t}$ . Therefore,  $\forall t \in [nT - 1, nT]$ , the market makers' estimation of  $H(\hat{m}_{nT}, nT)$  is

$$\Lambda_t = \mathbb{E} \left[ H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right] = \mathbb{E} \left[ e^{\log H(\hat{m}_{nT}, nT)} | \mathcal{F}_t^Y \right] = e^{\Lambda_t^* + \frac{1}{2} \Sigma_{\Lambda^*,t}} = e^{\Lambda_t^* + \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 (nT - t) \sigma_v^2}.$$

From Ito's Lemma,

$$\begin{aligned}
\frac{d\Lambda_t}{\Lambda_t} &= \frac{1}{\Lambda_t} \left[ \Lambda_t d\Lambda_t^* + \frac{1}{2} \Lambda_t (d\Lambda_t^*)^2 - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 \Lambda_t dt \right] \\
&= d\Lambda_t^* = \frac{-\gamma^A}{\beta - \gamma^A} d\hat{v}_t = -\frac{\gamma^A}{\beta} \lambda d\hat{Y}_t.
\end{aligned} \tag{A30}$$

Therefore, both  $V_t$  and  $\Lambda_t$  are functions of the adjusted order flow  $\hat{Y}_t$ . From the definition of price dynamics in equation (14),

$$P_t = \frac{\mathbb{E} \left[ H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right]}{\mathbb{E} \left[ H(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right]} = \frac{V(t, \hat{Y}_t)}{\Lambda(t, \hat{Y}_t)},$$

the equilibrium pricing rule is also a function of the adjusted order flow, i.e.,  $P(t, \hat{Y}_t)$ .

I apply Ito's Lemma to  $V_t$ ,

$$\frac{dV_t}{V_t} = \frac{d(P_t \Lambda_t)}{P_t \Lambda_t} = \frac{dP_t}{P_t} + \frac{d\Lambda_t}{\Lambda_t} + \frac{dP_t d\Lambda_t}{P_t \Lambda_t}. \quad (\text{A31})$$

From equations (A28) and (A30), I find

$$\frac{dP(t, \hat{Y}_t)}{P(t, \hat{Y}_t)} = \lambda d\hat{Y}_t + \gamma^A \beta \Delta Q dt, \quad \text{with } P_{nT-1} = e^{\mu_P - \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 + \frac{1}{2} \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2}. \quad (\text{A32})$$

Furthermore, from equation (A23), the process  $\hat{Y}_t$  is a Brownian bridge with instantaneous variance  $\sigma_z^2$  with respect to the insider's filtration, terminating at  $(\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \mu_V) / \left(\frac{\beta - \gamma^A}{\beta} \lambda\right)$  (Karatzas and Shreve (1987)). It satisfies  $\hat{Y}_t \rightarrow (\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \mu_V) / \left(\frac{\beta - \gamma^A}{\beta} \lambda\right)$  with probability 1 as  $t \rightarrow nT$ .<sup>52</sup> This implies the equilibrium price in equation (A32) satisfies :

$$\begin{aligned} \log P_t &= \log P_{nT-1} + \lambda \hat{Y}_t - \left[ \frac{1}{2} \sigma_v^2 - \gamma^A \beta \Delta Q \right] (t - (nT - 1)) \\ &= \log P_{nT-1} + \lambda \hat{Y}_t - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 (t - (nT - 1)) \\ &\rightarrow \beta \hat{m}_{nT-1} - \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 + N(nT) + \frac{1}{2} \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2 + \beta (\hat{m}_{nT} - \hat{m}_{nT-1}) - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 \\ &\rightarrow \beta \hat{m}_{nT} + N(nT) = \log A(\hat{m}_{nT}, nT). \end{aligned} \quad (\text{A33})$$

almost surely as  $t \rightarrow nT$  from the insider's information. This is equivalent to  $P_t \rightarrow A(\hat{m}_{nT}, nT)$  with probability 1 as  $t \rightarrow nT$  under the insider's filtration. ■

## B.2 Step 2: Insider's Optimal Strategy

Second, I capture the insider's optimal trading strategy when the equilibrium pricing rule is a function of the adjusted order flow, i.e.,  $P(t, \hat{Y}_t)$ .

**Proof of Lemma 3.** By Theorem 7.6 in Chapter 5 of Karatzas and Shreve (1991) (Feynman-Kac representation), the value function  $J$  defined in equation (26), is a unique solution to the Bellman equation (24) with the terminal condition  $J(nT, y, A(\hat{m}_{nT}, nT)) = j(y, A(\hat{m}_{nT}, nT))$ .

<sup>52</sup>The distribution of a Brownian bridge is the same as a Brownian motion conditional on the terminal value being known.  $(\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \mu_V) / \left(\frac{\beta - \gamma^A}{\beta} \lambda\right)$  is the terminal value of  $\hat{Y}_t$ , which is normally distributed with mean zero and variance  $\sigma_z^2$  and is independent of  $Z$ . Hence, the distribution of  $\hat{Y}_t$ , unconditional on the terminal value or  $Z$  (i.e., from the market makers' filtration), are the distribution of a Brownian motion with variance  $\sigma_z^2$ . This is consistent with what I get from the filtering theory in equation (A25).

Taking the derivative under the expectation operator yields<sup>53</sup>

$$\begin{aligned} J_y(t, y, A(\hat{m}_{nT}, nT)) &= \mathbb{E} [j_y(y + \omega_{nT} - \omega_t, A(\hat{m}_{nT}, nT))] \\ &= \mathbb{E} [g(y + \omega_{nT} - \omega_t)] - A(\hat{m}_{nT}, nT) \\ &= P(t, y) - A(\hat{m}_{nT}, nT), \end{aligned}$$

which shows  $J(t, y, A(\hat{m}_{nT}, nT))$  also satisfy equation (23) with  $P(t, y)$  as defined by (25). ■

**Proof of Lemma 4.** For any trading strategy  $\theta_t$ , apply Ito's Lemma to the value function,

$$\begin{aligned} J(nT, \hat{Y}_{nT}, A(\hat{m}_{nT}, nT)) &= J(nT-1, \hat{Y}_{nT-1}, A(\hat{m}_{nT}, nT)) + \int_{nT-1}^{nT} \left\{ J_t dt + J_y d\hat{Y}_t + \frac{1}{2} J_{yy} (d\hat{Y}_t)^2 \right\} \\ &= J(nT-1, \hat{Y}_{nT-1}, A(\hat{m}_{nT}, nT)) + \int_{nT-1}^{nT} \left\{ J_t dt + J_y ([\theta_t - \hat{\theta}_t] dt + dZ_t) + \frac{1}{2} \sigma_z^2 J_{yy} \right\} \\ &= J(nT-1, \hat{Y}_{nT-1}, A(\hat{m}_{nT}, nT)) - \int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P(t, \hat{Y}_t)) (\theta_t dt + dZ_t), \end{aligned}$$

where I use equations (23) and (24). I can rearrange this as

$$\begin{aligned} \int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P(t, \hat{Y}_t)) \theta_t dt &= J(nT-1, \hat{Y}_{nT-1}, A(\hat{m}_{nT}, nT)) - J(nT, \hat{Y}_{nT}, A(\hat{m}_{nT}, nT)) \\ &\quad - \int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P(t, \hat{Y}_t)) dZ_t \end{aligned}$$

The left-hand side is the profit of the insider, and the right-hand side is bounded above by

$$J(nT-1, \hat{Y}_{nT-1}, A(\hat{m}_{nT}, nT)) - \int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P(t, \hat{Y}_t)) dZ_t \quad (\text{A34})$$

due to the nonnegativity of  $J(nT, \hat{Y}_{nT}, A(\hat{m}_{nT}, nT))$  in equation (26). The no-double-strategies condition

$$E \int_{nT-1}^{nT} P_t^2 dt < \infty$$

implies that the stochastic integral in (A34) has a zero expectation. Therefore,

$$E \int_{nT-1}^{nT} \{ [A(\hat{m}_{nT}, nT) P(t, \hat{Y}_t)] \theta_t dt \} \leq J(nT-1, P_{nT-1}, A(\hat{m}_{nT}, nT)),$$

with equality if and only if  $\hat{Y}_{nT} = g^{-1}(A(\hat{m}_{nT}, nT))$ , which is equivalent to  $P(nT, \hat{Y}_{nT}) = A(\hat{m}_{nT}, nT)$  from equation (25). Thus,  $J(nT-1, \hat{Y}_{nT-1}, A(\hat{m}_{nT}, nT))$  is an upper bound on the insider's expected profit, conditional on the termination value  $A(\hat{m}_{nT}, nT)$ , and the upper bound is realized - and the corresponding strategy is consequently optimal - if and only if  $P(nT, \hat{Y}_{nT}) = A(\hat{m}_{nT}, nT)$ . ■

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<sup>53</sup>The proof that the derivative of the right-hand side of (26) can be taken under the expectation operator is similar to Back (1992).

Having established these results, finally, I show that the conjectured rule by the market makers is indeed consistent with the insider's optimal choice, as stated in Theorem 1.

**Proof of Theorem 1.** Since  $\hat{Y}_{nT} = g^{-1}(A(\hat{m}_{nT}, nT))$  a.s., for any scalar  $a$ , the probability, given the market makers' information at time  $nT - 1$ , that  $\hat{Y}_{nT} \leq a$  is  $F(g(A(\hat{m}_{nT}, nT)))$  where  $F$  is the distribution function of  $A(\hat{m}_{nT}, nT)$ . According to Lemma , the distribution function of  $\hat{Y}_{nT}$ , given the market makers' information at time 0, is normal distribution with mean zero and variance  $\sigma_z^2$  and I denote it as  $N$ . Therefore,  $N = F \circ g$ , implying  $g = F^{-1} \circ N$ . When  $\log A(\hat{m}_{nT}, nT)$  is normally distributed with mean  $\beta\hat{m}_{nT-1} + N(nT)$  and variance  $\beta^2 [q_{nT-1} - q_{nT}] = \sigma_v^2$ . Set  $g(y) = F^{-1}(N(y))$ :

$$F(g(y)) = N^* \left( \frac{\log g(y) - [\beta\hat{m}_{nT-1} + N(nT)]}{\sigma_v} \right) = N^* \left( \frac{\hat{Y}_t}{\sigma_z} \right),$$

so

$$g(y) = \exp(\beta\hat{m}_{nT-1} + N(nT) + \lambda y), \quad (\text{A35})$$

where  $\lambda = \frac{\sigma_v}{\sigma_z}$  and  $g(y)$  is a increasing function in  $y$  since  $\lambda > 0$ . From the conjectured trading strategy in equation (15),  $\hat{\theta}_t \equiv \mathbb{E}[\theta_t | \mathcal{F}_t^Y] = \frac{\gamma^A \beta \Delta Q}{\lambda}$ . It implies

$$\begin{aligned} P(t, \hat{Y}_t) &= \mathbb{E} [g(\hat{Y}_t + \omega_{nT} - \omega_t)] \\ &= \mathbb{E} \left[ \exp \left( \beta\hat{m}_{nT-1} + N(nT) + \lambda \left( \hat{Y}_t + Z_{nT} - Z_t - \frac{\gamma^A \beta \Delta Q}{\lambda} (nT - t) \right) \right) \right] \\ &= \exp \left( \beta\hat{m}_{nT-1} + N(nT) + \lambda \hat{Y}_t + \frac{1}{2} \sigma_v^2 (nT - t) - \gamma^A \beta \Delta Q (nT - t) \right) \\ &= \exp \left( \log P_{nT-1} + \lambda \hat{Y}_t - \left[ \frac{1}{2} \sigma_v^2 - \gamma^A \beta \Delta Q \right] (t - (nT - 1)) \right) \end{aligned} \quad (\text{A36})$$

where  $P_{nT-1} = e^{\beta\hat{m}_{nT-1} - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 + N(nT) + \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2}$ , which is exactly equation (A33). Therefore, the pricing function in equation (A35) implies the price dynamics follow equation (A32).

Equation (25) implies that  $P(t, \omega_t)$  is a martingale under the filtration generated by  $\omega$ . This implies the price dynamics and the expected trading volume  $\hat{\theta}(t)$  with respect to  $\mathcal{F}_t$  must satisfy

$$P_t - \hat{\theta}(t) P_y + \frac{1}{2} \sigma_z^2 P_{yy} = 0. \quad (\text{A37})$$

It's very straightforward to show this pricing rule in equation (A36) satisfies the above property.

Besides, In the proof of Lemma 2, I have already shown that the trading strategy in (28) implies  $P(t, \hat{Y}_t) \rightarrow A(\hat{m}_{nT}, nT)$  with probability 1 as  $t \rightarrow nT$ . It follows that the strategy (28) is optimal. Therefore,  $\{P_t, \theta_t\}$  in equations (27) and (28) is an equilibrium.

Combining equations (26) and (A35), the maximized expected profit of the insider is

$$\begin{aligned} J(t, P(t, \hat{Y}_t), A(\hat{m}_{nT}, nT)) &= \frac{1}{2} \sigma_v \sigma_z (nT - t) A(\hat{m}_{nT}, nT) \\ &+ \frac{P(t, \hat{Y}_t) - A(\hat{m}_{nT}, nT) + A(\hat{m}_{nT}, nT) [\log A(\hat{m}_{nT}, nT) - \log P(t, \hat{Y}_t)]}{\lambda}. \end{aligned} \quad (\text{A38})$$

As in Back (1992), I explicitly indicate the conditional expectation at time  $t$  given the market makers' information by  $E^M[\cdot]$  and the conditional expectation given the insider's information by  $E^I[\cdot]$ . Given equation (A35), the pricing rule in equation (25) yields

$$\begin{aligned}
P(t, Z_t) &= E^I[g(Z_t + \omega_{nT} - \omega_t) | Z_t] \\
&\stackrel{(A35)}{=} E^I\left[\exp\left(\beta \hat{m}_{nT-1} + N(nT) + \lambda\left(Z_{nT} - \frac{\gamma^A \beta \Delta Q}{\lambda}(nT - t)\right)\right) | Z_t\right] \\
&= E^I[P(nT, Z_{nT}) | Z_t] \exp\left(-\gamma^A \beta \Delta Q(nT - t)\right), \tag{A39}
\end{aligned}$$

where the last equality comes from equation (A36) when  $t = nT$ :

$$\begin{aligned}
P(nT, Z_{nT}) &= \exp\left(\log P_{nT-1} + \lambda Z_{nT} - \left[\frac{1}{2}\sigma_v^2 - \gamma^A \beta \Delta Q\right]\right) \\
&= \exp(\beta \hat{m}_{nT-1} + N(nT) + \lambda Z_{nT}).
\end{aligned}$$

Rearrange equation (A39), I find

$$P(t, Z_t) \exp\left(-\gamma^A \beta \Delta Q(t - (nT - 1))\right) = E^I[P(nT, Z_{nT}) | Z_t] \exp\left(-\gamma^A \beta \Delta Q\right),$$

which implies  $P(t, Z_t) \exp\left(-\gamma^A \beta \Delta Q(t - (nT - 1))\right)$  is a martingale under the insider's information set. Since the distribution of  $Z_t$  with respect to the insider's information is the same as the distribution of  $\hat{Y}_t$  with respect to the market makers' information,

$$\begin{aligned}
P(t, \hat{Y}_t) \exp\left(-\gamma^A \beta \Delta Q(t - (nT - 1))\right) &= E^M[P(nT, \hat{Y}_{nT}) | \hat{Y}_t] \exp\left(-\gamma^A \beta \Delta Q\right) \\
&= E^M\left[P(nT, \hat{Y}_{nT}) | (\hat{Y}_s)_{s \leq t}\right] \exp\left(-\gamma^A \beta \Delta Q\right)
\end{aligned}$$

where the last equality using the Markov property of a Brownian motion. This implies

$$P(t, \hat{Y}_t) \exp\left(-\gamma^A \beta \Delta Q(t - (nT - 1))\right)$$

is a martingale under the insider's information set. This is equivalent to say  $P(t, \hat{Y}_t)$  is a submartingale with a deterministic growth rate  $\gamma^A \beta \Delta Q$  per unit of time since both  $\gamma^A$  and  $\beta$  are strictly positive. Similar argument applies to the price-response function  $P_{\hat{Y}}(t, \hat{Y})$ , which is also a submartingale with a deterministic growth rate  $\gamma^A \beta \Delta Q$  per unit of time.

Therefore, the unconditional expected return for any  $t \in [nT - 1, nT]$  is

$$\log \mathbb{E}\left[\frac{P_t}{P_{nT-1}} | \mathcal{F}_{nT-1}^Y\right] = \gamma^A \beta \Delta Q(t - (nT - 1)) = \gamma^A \beta \Delta Q(t - (nT - 1)), \tag{A40}$$

which implies the expected pre-FOMC announcement drift grows at a constant rate  $\gamma^A \beta \Delta Q$ . ■

Next, I prove the properties of the equilibrium in Theorem 1.

**Proof of Proposition 1.** In the meantime, the posterior variance of  $\log P_{nT}$  at time  $t \in [nT - 1, nT]$  is

$$\begin{aligned}
\text{Var} \left[ \log P_{nT} | \mathcal{F}_t^Y \right] &= \text{Var} \left[ \log A(\hat{m}_{nT}, nT) | \mathcal{F}_t^Y \right] \\
&= \text{Var} \left\{ \log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] - \log [H(\hat{m}_{nT}, nT)] | \mathcal{F}_t^Y \right\} \\
&= \text{Var} \left\{ \log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)] | \mathcal{F}_t^Y \right\} + \text{Var} \left\{ \log [H(\hat{m}_{nT}, nT)] | \mathcal{F}_t^Y \right\} \\
&\quad - 2\text{Cov} \left( \log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)], \log [H(\hat{m}_{nT}, nT)] | \mathcal{F}_t^Y \right) \\
&= \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 (nT - t) - 2 \frac{-\gamma^A (\beta - \gamma^A)}{\beta^2} \sigma_v^2 (nT - t) + \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 (nT - t) \\
&= \sigma_v^2 (nT - t) = \beta^2 \Delta Q (nT - t).
\end{aligned}$$

Therefore, the reduction of the uncertainty at time  $t$  comparing to  $nT - 1$  is

$$\text{Var} \left[ \log P_{nT} | \mathcal{F}_t^Y \right] - \text{Var} \left[ \log P_{nT} | \mathcal{F}_{nT-1}^Y \right] = \beta^2 \Delta Q [(nT - 1) - t],$$

which implies the uncertainty reduces at a constant rate  $\beta^2 \Delta Q$  per unit of time.  $\blacksquare$

To prove Proposition 2, I first characterize the equilibrium when the market makers are risk-neutral, i.e., the original Kyle model under  $\gamma_{\text{Kyle}}^A = 0$ .

**Lemma 6.** *When the market makers are risk-neutral,  $\forall t \in [nT - 1, nT]$ , there exists an equilibrium where the price process  $P_t^{\text{Kyle}}$  and optimal strategy of the insider  $\theta_t^{\text{Kyle}}$  have dynamics,*

$$\frac{dP^{\text{Kyle}}(t, Y_t^{\text{Kyle}})}{P^{\text{Kyle}}(t, Y_t^{\text{Kyle}})} = \lambda dY_t^{\text{Kyle}} \quad \text{with } P_{nT-1}^{\text{Kyle}} = e^{\mu_P + \frac{1}{2}\sigma_v^2}, \quad (\text{A41})$$

$$\theta^{\text{Kyle}}(t, Y_t^{\text{Kyle}}) = \frac{\log [A(\hat{m}_{nT}, nT)] - \mu_P}{(nT - t) \lambda} - \frac{Y_t^{\text{Kyle}}}{nT - t}, \quad (\text{A42})$$

where  $\mu_P$ ,  $\sigma_v$ , and  $\lambda$  are defined in Lemma 2. Here the dynamics of aggregate order flow  $Y^{\text{Kyle}}$  is the sum of the insider's demand the liquidity traders' demand:

$$dY_t^{\text{Kyle}} = \theta_t^{\text{Kyle}} dt + dZ_t.$$

**Proof.** See Back (1992) or let  $\gamma^A = 0$  in Theorem 1.  $\blacksquare$

**Proof of Proposition 2.** It is a straightforward calculation from the equations (27), (28), (A41) and (A42).  $\blacksquare$

## C Proof of Theorem 2

The proof is in several steps. At the beginning, I provide the essential tools to construct the equilibrium of the model.<sup>54</sup>

### C.1 Step 0: tools for the market makers' updating

**Lemma 7.** Let  $\mu(t, V)$  be the estimate of the unnormalized density function of the random variable  $V = H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  given the stochastic differential equation (38) when the insider is informed. Then  $\mu(t, V)$  must satisfy the following stochastic differential equation (Zakai equation):

$$d\mu(t, V) = \frac{\theta(t, \tilde{V})}{\sigma_z^2} \mu(t, V) dY_t, \quad \mu(0, V) = f(V),$$

which has a unique solution

$$\mu(t, V) = f(V) \exp \left[ \frac{1}{\sigma_z^2} \left( \int_0^t \theta(s, V) dY(s) - \frac{1}{2} \int_0^t \theta^2(s, V) ds \right) \right].$$

Hence, the value estimate  $V(t)$  of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  is given by

$$V(t) \equiv \mathbb{E} [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}] = \frac{\int_V V \mu(V, t) dV}{\int_V \mu(V, t) dV} \quad (\text{A43})$$

where  $f(v) = \frac{dF(v)}{dv}$  is the prior probability density function at time 0.

**Proof.** See Zakai (1969) or Baras (1991). ■

**Lemma 8.** The value estimate given by (A43) satisfies the stochastic differential equation

$$dV(t) = \lambda(t) (dY_t - \hat{\theta}(t) dt), \quad (\text{A44})$$

where

$$\hat{\theta}(t) = \mathbb{E} [\theta(t, \tilde{V}) | \mathcal{F}_{1,t}] = \frac{\int_V \theta(t, \tilde{V}) \mu(V, t) dv}{\int_V \mu(V, t) dv} \quad (\text{A45})$$

and

$$\lambda(t) \equiv \frac{\mathbb{E} [\theta(t, \tilde{V}) | \mathcal{F}_{1,t}] - V(t) \hat{\theta}(t)}{\sigma_z^2}. \quad (\text{A46})$$

In addition,

$$\hat{Y}_{1,t} \equiv Y_t - \int_0^t \hat{\theta}(s) ds \quad (\text{A47})$$

is a Brownian Motion with instant variance  $\sigma_z^2$  under  $\mathcal{F}_{1,t}$ .

**Proof.** Applying Ito's Lemma to equation (A43) leads to the above standard filtering results. ■

<sup>54</sup>The method of proof is based on Li (2013) that solves the economy with the risk-neutral market makers. He applies the "sequential detection" in the filtering literature.

Through observing the aggregate trading volume  $Y_t$ , market makers estimate the probability that  $Y_t$  is generated by the the insider has private information or not. This updating problem can be solved as to calculate the likelihood ratio between the two hypotheses,  $\delta = 1$  versus  $\delta = 0$ . Following Li (2013), the logarithm of the likelihood ratio between hypotheses (38) and (39) is given by

$$\phi(t) \equiv \frac{1}{\sigma_z^2} \left( \int_0^t [\hat{\theta}(s) - \theta(s, \bar{V})] dY(s) - \frac{1}{2} \int_0^t [\hat{\theta}^2(s) - \theta^2(s, \bar{V})] ds \right)$$

where  $\hat{\theta}$  is as defined by (A45).

**Lemma 9.** *The market makers' estimate of the probability that the strategic trader has private information*

$$\pi(t) = \mathbb{E} \left[ \delta | \mathcal{F}_t^Y \right] = \frac{\pi_{nT-1} \exp[\phi(t)]}{1 - \pi_{nT-1} + \pi_{nT-1} \exp[\phi(t)]} \quad (\text{A48})$$

satisfies the following stochastic differential equation:

$$d\pi(t) = \frac{\pi(t) [1 - \pi(t)]}{\sigma_z^2} (\hat{\theta}(t) - \theta(t, \bar{V})) d\hat{Y}(t), \quad \pi(0) = \pi_{nT-1} \quad (\text{A49})$$

where

$$\hat{Y}(t) = Y_t - \int_0^t (\pi(s) \hat{\theta}(s) + [1 - \pi(s)] \theta(s, \bar{V})) ds \quad (\text{A50})$$

is the information process, which is a Brownian motion with instantaneous variance  $\sigma_z^2$  under the filtration  $\mathcal{F}_t^Y$ .

**Proof.** The definition of  $\pi(t)$  in equation (A48) is obtained by the Bayes' rule. By Ito's Lemma,

$$\begin{aligned} d\pi(t) &= \pi_\phi d\phi + \frac{1}{2} \pi_{\phi\phi} (d\phi)^2 \\ &= \pi(1 - \pi) d\phi + \frac{1}{2} \pi(1 - \pi)(1 - 2\pi) (d\phi)^2 \\ &= \frac{\pi(t) [1 - \pi(t)]}{\sigma_z^2} (\hat{\theta}(t) - \theta(t, \bar{V})) d\hat{Y}(t) \end{aligned}$$

where

$$\hat{Y}(t) = Y_t - \int_0^t (\pi(s) \hat{\theta}(s) + [1 - \pi(s)] \theta(s, \bar{V})) ds.$$

■

Lemma 9 shows that the market makers' probability estimate is governed by a nonlinear stochastic differential equation. Note that when the prior  $\pi_{nT-1} = 0$  or  $\pi_{nT-1} = 1$ , the solution to the belief dynamics (A49) is  $\pi(t) \equiv 0$  or  $\pi(t) \equiv 1$ , respectively.



## C.2 Step 1: market makers' updating

First, I show that given the insider's trading strategies when she is informed and not informed, how the market makers estimate the probability that the insider has private information and the price dynamics through nonlinear filtering.

Let  $\Pi(t, y)$  be an arbitrary function in  $C^{1,2}$  on  $[0, 1] \times R$  with a close range  $[0, 1]$ . At time  $nT - 1$ ,  $\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]$  is normally distributed with mean  $\mu_v$  and variance  $\left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2$ . I define

$$h(y) = \exp\left(\mu_v + \frac{\beta - \gamma^A}{\beta} \lambda y\right) \text{ and } \bar{V} = e^{\mu_v + \frac{1}{2} \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2}. \text{ This implies } h^{-1}(\bar{V}) = \frac{\beta - \gamma^A}{2\lambda} \sigma_v^2.$$

I guess the insider's trading strategy follows

$$\theta(t, y, V) = \frac{h^{-1}(V) - h^{-1}(\bar{V}) - \Pi(t, y) [y - h^{-1}(\bar{V})]}{nT - t} + \bar{\theta}(t, y), \quad (\text{A51})$$

and

$$\Theta(t, y) = \frac{[1 - \Pi(t, y)] [y - h^{-1}(\bar{V})]}{nT - t} + \bar{\theta}(t, y). \quad (\text{A52})$$

The expected trading rate of the insider under market makers' perspective  $\mathcal{F}_t^Y$  is

$$\bar{\theta}(t, y) = \frac{\left(\frac{\gamma^A}{\beta} \lambda \sigma_z^2 \Pi(t, y) - \Pi_y(t, y) \sigma_z^2\right) \cdot E(t, y) + \Pi_y(t, y) \sigma_z^2}{\Pi \cdot E(t, y) + 1 - \Pi}, \quad (\text{A53})$$

where I let  $E(t, y)$  be defined as<sup>55</sup>

$$E(t, y) = e^{-\frac{\gamma^A}{\beta} \lambda y - \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (t - (nT - 1))}. \quad (\text{A54})$$

The following Lemma states the market makers' expectation of the insider's order rate and their value estimate of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$ , given the insider's order rate  $\theta(t, y, V)$  defined in equation (A51).

**Lemma 10.** *Let  $\hat{Y}_{1,t}$  be a Brownian bridge that satisfies*

$$d\hat{Y}_{1,t} = [\theta(t, \hat{Y}_{1,t}, V) - \Theta(t, \hat{Y}_{1,t})] dt + dZ_t \quad (\text{A55})$$

$$= \frac{h^{-1}(V) - \hat{Y}_{1,t}}{nT - t} dt + dZ_t \quad (\text{A56})$$

with  $\hat{Y}_{1, nT-1} = 0$ . If the insider's order rate is  $\theta(t, \hat{Y}_{1,t}, V)$ , as defined by (A51),  $\Theta(t, \hat{Y}_{1,t})$  as defined by (A52), is then the market makers' expected order rate from the insider, conditional on the insider having private information. That is,

$$\hat{\theta}(t) = \mathbb{E}[\theta(t, \hat{Y}_{1,t}, V) | \mathcal{F}_{1,t}] = \Theta(t, \hat{Y}_{1,t}).$$

<sup>55</sup>As shown later, Li (2013) is a special case of this economy where  $\bar{\theta}(t, y) = 0$  when the market makers are risk-neutral.

Furthermore, the expected value of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  under  $\mathcal{F}_{1,t}$  is

$$V(t) = \mathbb{H}(t, \hat{Y}_{1,t}),$$

where

$$\mathbb{H}(t, y) = \mathbb{E}[h(y + Z_{nT} - Z_t)],$$

where the expectation is taken over the Brownian motion  $Z$ .

**Proof.** See Lemma 6 in Li (2013). ■

From equation (A49), the market makers' estimate of the probability that the insider has private information satisfies

$$\begin{aligned} d\Pi(t, \hat{Y}_{1,t}) &= \frac{\Pi(t, \hat{Y}_{1,t}) [1 - \Pi(t, \hat{Y}_{1,t})] \hat{Y}_{1,t} - h^{-1}(\bar{V})}{\sigma_z^2} \frac{\hat{Y}_{1,t} - h^{-1}(\bar{V})}{nT - t} \\ &\times \left( d\hat{Y}_{1,t} + \frac{[1 - \Pi(t, \hat{Y}_{1,t})] [\hat{Y}_{1,t} - h^{-1}(\bar{V})]}{nT - t} dt \right) \end{aligned} \quad (\text{A57})$$

with  $\Pi(nT - 1, \hat{Y}_{1,nT-1}) = \pi_{nT-1}$ . This equation holds because of Lemma 9 and the following equation:

$$\begin{aligned} dY_t &\stackrel{(\text{A50}), (\text{A61})}{=} d\hat{Y}(t) + \bar{\theta}(t, y) \\ &\stackrel{(\text{A47})}{=} d\hat{Y}_{1,t} + \Theta(t, y) \\ &\stackrel{(\text{A52})}{=} d\hat{Y}_{1,t} + \frac{[1 - \Pi(t, y)] [y - h^{-1}(\bar{V})]}{nT - t} + \bar{\theta}(t, y). \end{aligned} \quad (\text{A58})$$

As shown in Li (2013),  $\forall t \in [nT - 1, nT]$ , the solution to the stochastic differential equation (A57) is:

$$\Pi(t, \hat{Y}_{1,t}) = \frac{\pi_{nT-1} \exp\left(\frac{1}{2\sigma_z^2} \frac{[\hat{Y}_{1,t} - h^{-1}(\bar{V})]^2}{nT - t} + \frac{1}{2} \log(nT - t) - \frac{[h^{-1}(\bar{V})]^2}{2\sigma_z^2}\right)}{1 - \pi_{nT-1} + \pi_{nT-1} \exp\left(\frac{1}{2\sigma_z^2} \frac{[\hat{Y}_{1,t} - h^{-1}(\bar{V})]^2}{nT - t} + \frac{1}{2} \log(nT - t) - \frac{[h^{-1}(\bar{V})]^2}{2\sigma_z^2}\right)}, \quad (\text{A59})$$

which is the market makers' optimal estimate of the probability that the insider has private information, given the insider's order rate  $\theta(t, \hat{Y}_{1,t}, V)$  defined by equation (A51).

When the insider is not better informed, Lemma (10) implies that her trading strategy follows

$$\theta(t, y, \bar{V}) = -\frac{\Pi(t, y) [y - h^{-1}(\bar{V})]}{nT - t} + \bar{\theta}(t, y),$$

which implies

$$\hat{\theta}(t) - \theta(t, y, \bar{V}) = \Theta(t, y) - \theta(t, y, \bar{V}) = \frac{y - h^{-1}(\bar{V})}{nT - t}, \quad (\text{A60})$$

and

$$\Pi(t, y) \Theta(t, y) + [1 - \Pi(t, y)] \theta(t, y, \bar{V}) = \bar{\theta}(t, y). \quad (\text{A61})$$

When the insider has no private information, I can rewrite the dynamics of the probability estimate as

$$d\Pi(t, \hat{Y}_{1,t}) = \frac{\Pi(t, \hat{Y}_{1,t}) [1 - \Pi(t, \hat{Y}_{1,t})] \hat{Y}_{1,t} - h^{-1}(\bar{V})}{\sigma_z^2} \frac{\hat{Y}_{1,t} - h^{-1}(\bar{V})}{nT - t} \times \left( dZ_t - \frac{\Pi(t, \hat{Y}_{1,t}) [\hat{Y}_{1,t} - h^{-1}(\bar{V})]}{nT - t} dt \right). \quad (\text{A62})$$

Therefore, conditional on whether the insider is informed or not, there are two different dynamics of probability estimation, as stated in equations (A57) and (A62).

As stated in Lemma 11, a direct application of Theorem 1 in Li (2013) leads to the same property of probability estimate.

**Lemma 11.** *Let  $\hat{Y}_{1,t}$  be the Brownian bridge as defined by equation (A56) for any  $V \in \mathcal{V}$ . Suppose that the prior  $\pi_{nT-1} \in (0, 1)$ . Then, the market makers' probability estimate that the insider has private information,  $\Pi(t, \hat{Y}_{1,t})$ , always resides in  $(0, 1)$  for all  $t < nT$ . Upon announcements, it converges to 1 or 0 depending on whether the insider has private information or not.*

Since  $\log [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)]$  is normally distributed with mean  $\mu_v$  and variance  $\left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2$  at time  $nT - 1$ ,  $h(y) = \exp\left(\mu_v + \frac{\beta - \gamma^A}{\beta} \lambda y\right)$  and  $\bar{V} = e^{\mu_v + \frac{1}{2} \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2}$ . From Lemma 10,  $\forall t \in [nT - 1, nT]$ , the estimation of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  conditional on  $\delta = 1$  follows

$$\begin{aligned} V(t) &= \mathbb{E} [H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}] \\ &= \mathbb{H}(t, \hat{Y}_{1,t}) = \mathbb{E} [h(\hat{Y}_{1,t} + Z_{nT} - Z_t)] \\ &= \exp\left(\mu_v + \frac{\beta - \gamma^A}{\beta} \lambda \hat{Y}_{1,t} + \frac{1}{2} \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2 (nT - t)\right), \end{aligned}$$

while the estimation of  $H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT)$  conditional on  $\delta = 0$  is  $\bar{V} = e^{\mu_v + \frac{1}{2} \left(\frac{\beta - \gamma^A}{\beta}\right)^2 \sigma_v^2}$ , where  $\mu_v = (\beta - \gamma^A) x_{nT-1} + N(nT)$ .

Similarly, the estimation of SDF  $H(\hat{m}_{nT}, nT)$  conditional on  $\delta = 1$  follows

$$\begin{aligned} \Lambda(t) &= \mathbb{E} [H(\hat{m}_{nT}, nT) | \mathcal{F}_{1,t}] \\ &= \exp\left(\mu_\Lambda - \frac{\gamma^A}{\beta} \lambda \hat{Y}_{1,t} + \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2 (nT - t)\right), \end{aligned}$$

while the estimation of SDF  $H(\hat{m}_{nT}, nT)$  conditional on  $\delta = 0$  is  $\bar{\Lambda} = e^{\mu_\Lambda + \frac{1}{2} \left(\frac{\gamma^A}{\beta}\right)^2 \sigma_v^2}$ , where  $\mu_\Lambda = -\gamma^A x_{nT-1} + \mathcal{H}(nT)$ .

Therefore, this implies the price defined in equation (36) depends only on the current adjusted

trading flow  $\hat{Y}_{1,t}$ , which follows

$$\begin{aligned}
P(t, \hat{Y}_{1,t}) &= \frac{\Pi(t, \hat{Y}_{1,t}) V(t, \hat{Y}_{1,t}) + (1 - \Pi(t, \hat{Y}_{1,t})) \bar{V}}{\Pi(t, \hat{Y}_{1,t}) \Lambda(t, \hat{Y}_{1,t}) + (1 - \Pi(t, \hat{Y}_{1,t})) \bar{\Lambda}} \\
&= \frac{\Pi(t, \hat{Y}_{1,t}) e^{\mu_V + \frac{\beta - \gamma^A}{\beta} \lambda \hat{Y}_{1,t} + \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 (nT - t)} + (1 - \Pi(t, \hat{Y}_{1,t})) e^{\mu_V + \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2}}{\Pi(t, \hat{Y}_{1,t}) e^{\mu_\Lambda - \frac{\gamma^A}{\beta} \lambda \hat{Y}_{1,t} + \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 (nT - t)} + (1 - \Pi(t, \hat{Y}_{1,t})) e^{\mu_\Lambda + \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2}} \\
&= P_{nT-1} \frac{\Pi(t, \hat{Y}_{1,t}) e^{\frac{\beta - \gamma^A}{\beta} \lambda \hat{Y}_{1,t} - \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 (t - (nT - 1))} + 1 - \Pi(t, \hat{Y}_{1,t})}{\Pi(t, \hat{Y}_{1,t}) e^{-\frac{\gamma^A}{\beta} \lambda \hat{Y}_{1,t} - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 (t - (nT - 1))} + 1 - \Pi(t, \hat{Y}_{1,t})}, \tag{A63}
\end{aligned}$$

where  $P_{nT-1} = e^{\beta \hat{m}_{nT-1} - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 + N(nT) + \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2}$ .

### C.3 Step 2: Insider's Optimal Strategy

In this section, I show that if the dynamics of price follows equation (A63), then the optimal trading strategy of the insider is indeed of the form given in equation (A51) through verification proof.

Given the market makers' pricing rule,  $P(t) = P(t, \hat{Y}_{1,t})$ , the insider chooses the order rate to maximize her trading profit. When the insider has private information, for each terminal value  $A(\hat{m}_{nT}, nT)$ , she maximizes the terminal profit

$$\int_{nT-1}^{nT} (A(\hat{m}_{nT}, nT) - P(s, \hat{Y}_1(s))) \theta_s ds.$$

When the insider is not bettered informed, given no new information coming before announcements, her best estimation of the asset value at  $\forall t \in [nT - 1, nT]$  is always

$$\begin{aligned}
\bar{v}^* &\equiv \mathbb{E} \left[ \frac{H(\hat{m}_{nT}, nT)}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1}^Y]} A(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1}^Y \right] \\
&= \frac{\mathbb{E}[H(\hat{m}_{nT}, nT) A(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1}^Y]}{\mathbb{E}[H(\hat{m}_{nT}, nT) | \mathcal{F}_{nT-1}^Y]} \equiv \frac{\bar{V}}{\bar{\Lambda}}, \tag{A64}
\end{aligned}$$

which is the same as market makers at  $t = nT - 1$ .

Under Assumption 1, the insider chooses the order rate to maximize the expectation of her terminal profit given the market makers' pricing rule  $P(t) = P(t, \hat{Y}_{1,t})$ :

$$J(t, y; A(\hat{m}_{nT}, nT), \pi_{nT-1}) = \max_{\theta_t \in \mathcal{A}} \mathbb{E} \left[ \int_t^{nT} (A(\hat{m}_{nT}, nT) - P(s, \hat{Y}_s)) \theta_s ds | \hat{Y}_{1,t} = y, A(\hat{m}_{nT}, nT) \right]$$

subject to

$$d\hat{Y}_t = [\theta(t) - \hat{\theta}(t)] dt + dZ_t, \tag{A65}$$

where  $A(\hat{m}_{nT}, nT) = \bar{v}^*$  when the insider is not informed as shown in equation (A64).

The principle of optimality implies the following Bellman equation

$$\max_{\theta_t \in \mathcal{A}} \left\{ (A(\hat{m}_{nT}, nT) - P(t, y)) \theta_t + J_t + J_y [\theta_t - \hat{\theta}(t)] + \frac{1}{2} \sigma_z^2 J_{yy} \right\} = 0 \quad (\text{A66})$$

where the subscripts denote the derivatives. The necessary conditions for having an optimal solution to the Bellman equation (A66) are

$$J_y = P(t, y) - A(\hat{m}_{nT}, nT) \quad (\text{A67})$$

$$J_t + \frac{1}{2} \sigma_z^2 J_{yy} - \hat{\theta}(t) J_y = 0. \quad (\text{A68})$$

Under these necessary conditions, a direct application of Li (2013) leads to the following results:

**Lemma 12.** *Suppose the expected order rate  $\hat{\theta}(t) = \Theta(t, \hat{Y}_{1,t})$ , where  $\hat{Y}_{1,t}$  is the adjusted order at  $t$ . Let  $\omega_t = y$  and suppose that the stochastic differential equation*

$$d\omega_s = dZ_s - \Theta(s, \omega_s) ds, \quad \forall nT \geq s \geq t \geq nT - 1$$

*has a unique solution, where  $Z_s$  is a Brownian motion with instant variance  $\sigma_z^2$ . If there exists a strictly monotone function  $g(\cdot)$  such that the pricing rule is*

$$P(t, y) = \mathbb{E}[g(\omega_{nT}) | \omega_t = y], \quad (\text{A69})$$

then

$$J(t, y; v, \pi_{nT-1}) = \mathbb{E}[j(v, \omega_{nT}) | \omega_t = y]$$

is a smooth solution to the Bellman equations (18) and (19), where

$$j(v, y) = \int_y^{g^{-1}(v)} [v - g(x)] dx \geq 0, \quad \forall (v, y)$$

**Lemma 13.** *Any continuous trading strategy that makes  $\lim_{t \rightarrow nT^-} P(t, \hat{Y}_{1,t}) = A(\hat{m}_{nT}, nT)$  is optimal, where  $P(t, y)$  is as defined by equation (A69).*

Equation (A69) implies that  $P(t, \omega_t)$  is a martingale under the filtration generated by  $\omega$ .<sup>56</sup> This implies the price dynamics with respect to  $\mathcal{F}_{1,t}$  must satisfy

$$P_t - \Theta(t, y) P_y + \frac{1}{2} \sigma_z^2 P_{yy} = 0.$$

Finally, I am ready to prove that  $(X_0, X_1, P, \Pi)$  is an equilibrium. The insider's trading strategy,  $X_{\delta,t}$ , satisfies  $X_0(t) = \int_{nT-1}^t \theta(s, \hat{Y}_1(s), \bar{V}) ds$  and  $X_1(t) = \int_{nT-1}^t \theta(s, \hat{Y}_1(s), V) ds$ , where  $\hat{Y}_1$  is the solution to the stochastic differential equation (A56).  $\Pi(t, y)$  and  $\theta(t, y, V)$  are defined by equations (A59) and (A51), respectively.

<sup>56</sup>Notice that due to the existence of the SDF, the pricing rule  $P(t)$  is no longer a martingale under market makers (unconditional) information set  $\mathcal{F}_t^Y$ . Though both  $V(t)$  and  $\Lambda(t)$  are martingales under  $\mathcal{F}_t^Y$ .

**Proof of Theorem 2.** Note that I have established that  $\Pi(t, \hat{Y}_{1,t})$  is the optimal probability estimate of market makers given the trading strategy in equation (A51). Then, I need to show that the price dynamics in equation (A63) is a legitimate pricing rule. That is,

[1]. The price rule defined above satisfies

$$P_t - \Theta(t, y) P_y + \frac{1}{2} \sigma_z^2 P_{yy} = 0, \quad (\text{A70})$$

[2].  $P(nT, \hat{Y}_{1,t})$  is an increasing function of  $\hat{Y}_{1,t}$ ; and

[3].  $\lim_{t \rightarrow nT} P(t, \hat{Y}_{1,t}) = A(\hat{m}_{nT}, nT)$  almost surely.

The first condition can be shown by direct calculation. For convenience, I let

$$P(t, \hat{Y}_{1,t}) \equiv P_{nT-1} \frac{A(t, \hat{Y}_{1,t})}{B(t, \hat{Y}_{1,t})}$$

where

$$A(t, \hat{Y}_{1,t}) = \Pi(t, \hat{Y}_{1,t}) e^{\frac{\beta - \gamma^A}{\beta} \lambda \hat{Y}_{1,t} - \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 (t - (nT - 1))} + 1 - \Pi(t, \hat{Y}_{1,t}),$$

and

$$B(t, \hat{Y}_{1,t}) = \Pi(t, \hat{Y}_{1,t}) e^{-\frac{\gamma^A}{\beta} \lambda \hat{Y}_{1,t} - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 (t - (nT - 1))} + 1 - \Pi(t, \hat{Y}_{1,t}).$$

In addition, I let  $D(t, y)$  be defined as

$$D(t, \hat{Y}_{1,t}) = e^{\frac{\beta - \gamma^A}{\beta} \lambda \hat{Y}_{1,t} - \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 (t - (nT - 1))}.$$

I also use the definition of  $E(t, y)$  from equation (A54).

The first-order conditions and second-order conditions of the price dynamics (A63) are

$$P_t = P_{nT-1} B^{-2} \left\{ \left[ \left( \Pi_t - \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 \Pi \right) D - \Pi_t \right] B - A \left[ \left( \Pi_t - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 \Pi \right) E - \Pi_t \right] \right\},$$

$$P_y = P_{nT-1} B^{-2} \left\{ \left[ \left( \Pi_y + \frac{\beta - \gamma^A}{\beta} \lambda \Pi \right) D - \Pi_y \right] B - A \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] \right\},$$

and

$$P_{yy} = -P_{nT-1} B^{-2} \cdot 2B^{-1} \left\{ \left[ \left( \Pi_y + \frac{\beta - \gamma^A}{\beta} \lambda \Pi \right) D - \Pi_y \right] B - A \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] \right\} \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] + P_{nT-1} B^{-2} \left\{ \left[ \left( \Pi_{yy} + 2 \frac{\beta - \gamma^A}{\beta} \lambda \Pi_y + \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \lambda^2 \Pi \right) D - \Pi_{yy} \right] B - A \left[ \left( \Pi_{yy} - 2 \frac{\gamma^A}{\beta} \lambda \Pi_y + \left( \frac{\gamma^A}{\beta} \right)^2 \lambda^2 \Pi \right) E - \Pi_{yy} \right] \right\}.$$

Put these derivatives into the following equation:

$$\begin{aligned}
& \frac{B^2}{P_{nT-1}} \left\{ P_t - (\Theta - \bar{\theta} + \bar{\theta}) P_y + \frac{1}{2} \sigma_z^2 P_{yy} \right\} \\
&= \left\{ \left[ \left( \Pi_t - \frac{1}{2} \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \sigma_v^2 \Pi \right) D - \Pi_t \right] B - A \left[ \left( \Pi_t - \frac{1}{2} \left( \frac{\gamma^A}{\beta} \right)^2 \sigma_v^2 \Pi \right) E - \Pi_t \right] \right\} - \\
& \quad (\Theta - \bar{\theta} + \bar{\theta}) \left\{ \left[ \left( \Pi_y + \frac{\beta - \gamma^A}{\beta} \lambda \Pi \right) D - \Pi_y \right] B - A \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] \right\} + \\
& \frac{1}{2} \sigma_z^2 \left\{ \left[ \left( \Pi_{yy} + 2 \frac{\beta - \gamma^A}{\beta} \lambda \Pi_y + \left( \frac{\beta - \gamma^A}{\beta} \right)^2 \lambda^2 \Pi \right) D - \Pi_{yy} \right] B - A \left[ \left( \Pi_{yy} - 2 \frac{\gamma^A}{\beta} \lambda \Pi_y + \left( \frac{\gamma^A}{\beta} \right)^2 \lambda^2 \Pi \right) E - \Pi_{yy} \right] \right\} \\
& \quad - B^{-1} \sigma_z^2 \left\{ \left[ \left( \Pi_y + \frac{\beta - \gamma^A}{\beta} \lambda \Pi \right) D - \Pi_y \right] B - A \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] \right\} \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] \\
&= \underbrace{\left[ \Pi_t - (\Theta - \bar{\theta}) \Pi_y + \frac{1}{2} \sigma_z^2 \Pi_{yy} \right]}_{\text{term (a)}} [B(D-1) - A(E-1)] + \underbrace{\left[ \sigma_z^2 \Pi_y - (\Theta - \bar{\theta}) \Pi \right]}_{\text{term (b)}} \left( \frac{\beta - \gamma^A}{\beta} \lambda DB + \frac{\gamma^A}{\beta} \lambda AE \right) \\
& \quad - \underbrace{\left( \bar{\theta} + B^{-1} \sigma_z^2 \left( \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right) \right)}_{\text{term (c)}} \left\{ \left[ \left( \Pi_y + \frac{\beta - \gamma^A}{\beta} \lambda \Pi \right) D - \Pi_y \right] B - A \left[ \left( \Pi_y - \frac{\gamma^A}{\beta} \lambda \Pi \right) E - \Pi_y \right] \right\}.
\end{aligned} \tag{A71}$$

Next, I will show all of term (a), (b), and (c) are zero under the trading strategy (A51). From equations (A57) and (A58),

$$d\Pi(t, \hat{Y}_{1,t}) = \frac{\Pi(t, \hat{Y}_{1,t}) [1 - \Pi(t, \hat{Y}_{1,t})]}{\sigma_z^2} \frac{\hat{Y}_{1,t} - h^{-1}(\bar{V})}{nT - t} d\hat{Y}(t),$$

which implies  $\Pi(t, \hat{Y}_{1,t})$  is a martingale under market makers' information set since  $\hat{Y}(t)$  is a Brownian motion under  $\mathcal{F}_t^Y$ . In addition,

$$\begin{aligned}
d\hat{Y}_{1,t} &= dY_t - \Theta(t, \hat{Y}_{1,t}) dt \\
&= [d\hat{Y}(t) + \bar{\theta}(t, \hat{Y}_{1,t}) dt] - \Theta(t, \hat{Y}_{1,t}) dt \\
&= d\hat{Y}(t) - [\Theta(t, \hat{Y}_{1,t}) - \bar{\theta}(t, \hat{Y}_{1,t})] dt,
\end{aligned} \tag{A72}$$

I have

$$\Pi_t - [\Theta(t, y) - \bar{\theta}(t, y)] \Pi_y + \frac{1}{2} \sigma_z^2 \Pi_{yy} = 0 \tag{A73}$$

This shows that term (a) in equation (A71) is always zero for any  $(t, \hat{Y}_{1,t})$ .

Moreover, from equation (A59),

$$\Pi_y = \Pi(1 - \Pi) \frac{y - h^{-1}(\bar{V})}{\sigma_z^2 (nT - t)}. \tag{A74}$$

Combining with equation (A52), I have

$$\sigma_z^2 \Pi_y - [\Theta(t, y) - \bar{\theta}(t, y)] \Pi = 0, \quad (\text{A75})$$

which implies term (b) in equation (A71) is always zero for any  $(t, \hat{Y}_{1,t})$ .

The definition of  $\bar{\theta}(t, y)$  directly indicates term (c) in equation (A71) is always zero for any  $(t, \hat{Y}_{1,t})$ . Therefore, the price dynamics satisfy (A70) in the condition [1], which holds for all states of nature.<sup>57</sup>

As  $\Pi(nT, \hat{Y}_{1,nT}) = 1$  when the insider is better informed by Lemma 11, I have

$$P(nT, \hat{Y}_{1,t}) = P_{nT-1} e^{\lambda \hat{Y}_{1,t} - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 (t - (nT-1))},$$

which increases in  $\hat{Y}_{1,t}$  since  $\lambda > 0$ . This verifies the condition [2].

From Lemma 11, when the insider is better informed,

$$\begin{aligned} \lim_{t \rightarrow nT} \log P(t, \hat{Y}_{1,t}) &= \log P_{nT-1} - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 + \lambda \lim_{t \rightarrow nT} \hat{Y}_{1,t} \\ &= \log P_{nT-1} - \frac{1}{2} \frac{\beta - 2\gamma^A}{\beta} \sigma_v^2 + \beta (\hat{m}_{nT} - \hat{m}_{nT-1}) \quad \text{a.s.} \\ &= \log A(\hat{m}_{nT}, nT) \quad \text{a.s.} \end{aligned}$$

The second equality holds since  $\hat{Y}_{1,t}$  is a Brownian bridge that converges to  $h^{-1}(V)$  almost surely. The third equality comes from the definition of  $P_{nT-1}$ . Therefore, the condition [3] also holds. ■

**Proof of Proposition 3.** When the insider is informed, from Lemma 11 and Theorem 2, I show  $P(t, \hat{Y}_{1,t})$  converges to the true asset value  $A(\hat{m}_{nT}, nT)$  almost surely when  $t \rightarrow nT$ . Since all the private information is eventually incorporated into the price, it implies there is no uncertainty left just upon announcements:

$$\text{Var} \left[ \log P_{nT-} | \mathcal{F}_{nT-}^Y \right] = 0. \quad (\text{A76})$$

While when the insider is not informed,  $P(t, \hat{Y}_{1,t})$  converges to the initial price  $P_{nT-1}$  almost surely when  $t \rightarrow nT$ . There is no uncertainty reduction upon announcements since the insider has no information other than what the market makers have at  $nT - 1$ :

$$\text{Var} \left[ \log P_{nT-} | \mathcal{F}_{nT-}^Y \right] = \text{Var} \left[ \log P_{nT-} | \mathcal{F}_{nT-1}^Y \right] = \beta^2 \Delta Q. \quad (\text{A77})$$

Therefore, when  $\eta$  fraction of insider that is informed across these FOMC announcements,

$$\log \mathbb{E} \left[ \frac{P_{nT-}}{P_{nT-1}} \right] = \eta \gamma^A \beta \Delta Q, \quad (\text{A78})$$

$$\mathbb{E} \left[ \text{Var} \left[ \log P_{nT-} | \mathcal{F}_{nT-}^Y \right] - \text{Var} \left[ \log P_{nT-} | \mathcal{F}_{nT-1}^Y \right] \right] = -\eta \beta^2 \Delta Q, \quad (\text{A79})$$

where the expectations are taken over all states of natural. ■

<sup>57</sup>Combining equations (A74) and (A53), I can derive  $\bar{\theta}(t, y)$  as in equation (43).